Form and Counterform in the Periodic Polyhedral Honeycombs

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Abstract

This third part of a series of papers investigates the ordering of the all-space-filling periodic polyhedral honeycombs that I have previously advanced, from the perspective of form and counterform. Each periodic polyhedral honeycomb can be differentiated into a subset of contiguous periodic polyhedra, the "form", while the leftover space constitutes another subset of contiguous periodic polyhedral, the "counterform". Together, they fill all space. In certain cases, the contiguity of a subset is through just neutral vertex, axial or transverse edge, or transverse polygon, rather than through neutral axial polygon or polyhedron; but the behavior is rigorous, and further confirms the legitimacy of the metaorder of these honeycombs that I have previously advanced. Form and counterform are interchangeable, depending on which is being attended to, as in the figure-background perception of psychology, which is often illustrated with the classic "faces or vases" illusion, also known as the Rubin vase. Nevertheless, each demonstrates parallel and consistent structure within each of the three symmetry classes that I previously describe. In all cases, form and counterform are intertwined with each other, dividing space into two. In just four cases, form and counterform are identical (though displaced). Within Classes II and III, the expansion/contraction sequences of honeycombs that I previously identify again apply, so for each sequence, its form exhibits an expansion/contraction sequence, whilst its counterform simultaneously exhibits a corresponding sequence, and these are consistent for each sequence in the class. Class III is the most accessible (to understanding), and both form and counterform are manifest in √1 orientation relationships of either Primary Polyhedra One or Two (PP1 or PP2), respectively, mediated by their respective Neutral Elements (NEs: NE1 or NE2). Class II is rather interesting, as different form/counterform configurations can be abstracted, depending on

whether the primary relationships considered are of √1, (limited) √2, or √3 orientation. In this consideration, it is apropos to distinguish the two PPs of each honeycomb into a major (larger) and minor (smaller) polyhedron. Class I is unique with just one honeycomb, so does not demonstrate a sequence; but its honeycomb also can be subdivided into two forms that are identical, but in this singular case that are enantiomorphs of each other. The worth of the paper lies in the description of the relationships that illuminate the fundamental structure of these honeycombs, both individually, and more importantly, as various instances of the same metaorder. In some profound sense, this metaorder reflects in some aspect the fundamental structure of empirical 3-D space. Through these descriptions, these configurations become conceivable, imageable, and practical. Form and counterform of the honeycombs and their sequences enable the structuring of interpenetrating but distinct spaces that might be engineered to provide controllable porous membrane surfaces that allow two domains to interact with one another through high surface area interfaces. Their geometry may have relevance to such diverse applications as tensile arrays in the marine environment and in the microgravity of Space (stressed by enclosing pneumatic envelope); chemical structures of hybrid compounds; new materials engineered at nanoscale; filters; artificial bone tissue, etc.

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