Mathematical Modeling of the 3:5:4 Initiative - Refocusing Tertiary EFL Resources in Korea

Robert C. Meurant
Director, Institute of Traditional Studies
Ph. +82-10-7474-6226 • Email: rmeurant@me.com • Homepage: http://web.me.com/rmeurant/Institute/Home.html

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Abstract

Korean society, recognizing the importance of English as a Global Language, places considerable emphasis on raising the general level of English ability. Korean tertiary students are required to study English as a Foreign Language (EFL), but critical real-world constraints emerge: significant numbers of Korean Learners of English (KLOE) are unmotivated and/or have low aptitude; and there are limited resources with which to teach students English. As an alternative to addressing the political dimension of these issues - which means continue to prove ineffectual in resolving the challenges faced - I advance the 3:5:4 Initiative, which provides for a more effective use of resources. The overall level of English in Korean society could be raised through improving educational output. Firstly a qualitative, and then a quantitative restructuring of relative resource deployment is made. The need for such optimization is evident as the global economy suffers a serious period of recession, while the quality of English demanded from Korean graduates rises. Working from first principles, a straightforward mathematical model is developed of the relative proportioning of deployment of resources. The efficacy of resource deployment under two alternative scenarios indicates significant potential gains in efficiency in educational value.

1. Introduction

Within Korea, tertiary education of KLOE (Korean Learners of English) by Native Speakers of English is characterized by three factors:

- In general, all tertiary students (university and college) are required to study English as a Foreign Language;
- It is evident that a significant portion of these students are either highly unmotivated to learn English, or have a very low aptitude to do so, or quite commonly both; and
- There are quite limited resources with which to teach students English.

I advance what I term the 3:5:4 Initiative. I propose that by a conscious restructuring of the deployment of resources, more effective use can be made of these finite existent resources, and the overall level of English in Korea can then be substantially improved. The need for such optimization becomes more pressing as the global economy suffers a serious period of recession, while English continues to be ever more in demand. Further, the quality of L2 (Second Language) English demanded from graduates rises, as employers become more discriminating in their requirements.

I deliberately avoid discussion of the political dimension of these issues. In part, this is because it has become apparent that politics alone is incapable of engendering satisfactory solutions. Dissatisfaction with the state of English as a Foreign Language (EFL) education in Korea has been expressed at the very highest governmental levels for a considerable period of time, but the problems are deep-seated and remain unresolved. It is apparent that this lack of resolution arises because the essential problem is not political, but structural, and is therefore not solvable by political means in isolation from objective analysis, revisioning, and a refocusing of resources.

As an alternative, I therefore develop from first principles a mathematical model of the deployment of resources and consider the likely efficacy of such deployment under two alternative scenarios. My remarks are limited to the teaching of English by native speakers of English, but the argument can be extended to include tertiary-level teaching of English in Korea by both non-native English-speaking

(predominantly Korean) teachers of English as well as by native English-speaking teachers of English. Further extension could be made from the tertiary, to the EFL educational system in general.

The argument often used to justify the status of tertiary-level English instruction as being all-but-mandatory in Korea is the "No Child Left Behind" policy in the United States [1], where English literacy is rightly regarded as a universal issue of great importance. However, I question just how appropriate this particular internal American policy is to English education in Korea, given that - in America - English is in general the *lingua franca,* but within Korea it is not. The deserved importance of English as a global language is widely recognized by Korean society, but the key differential I wish to draw attention to is whether the laudable goal - of wide-spread English literacy in Korean society - is best achieved through an egalitarian attempt to concentrate resources in more-or-less equal measure on the English-speaking abilities of each and every tertiary student; or whether it might better be achieved by a thoughtful refocusing of resources, so that certain areas of English education receive preferential treatment, while other areas are deliberately allocated less. The discipline of Structural Morphology - the study of the abstract nature of form in Nature and in human endeavor - suggests that, in general, decentralized egalitarian deployments of resources (as are currently applied) are simply inefficient when compared with a degree of centralization. In sum, complexity benefits from - and may even require - a measure of informed specialization.

II. The Relevance of Structural Morphology

I draw my inspiration on this matter from the field of Structural Morphology (which use differs markedly from the widespread use in linguistics of the term 'morphology'). Here, structural morphology refers to the nature of form, which may be approached through logical, numerical, spatial, and temporal structure. Inherent in this perspective is the recognition of natural pattern - form is discerned in phenomena (and sometimes also in noumena), rather than being imposed from without. This gives rise to the idea of natural form, structure or pattern that is considered to characterize the natural world and the cosmos, which ideas I elsewhere pursue in a work on Natural Harmony [2]. Critically, reality is recognized to extend well beyond and even to underlie the mere socio-political construction of it.

Such natural pattern is readily discernable in exemplars such as the Periodic Table of Elements; the Regular and Semi-regular Polyhedra [3] (a symmetry set derived from my own research being shown in Fig. 1 above); the Regular and Semi-regular tilings of the plane [4]; the harmonic structure of rational number [5]; and natural proportioning systems observed in nature and historically employed for their evident efficacy and aesthetic beauty [6] - such as the Pythagorean Triplets {3:4:5}, {5:12:13}, {7:24:25}... as schematized below in Fig. 3, and, most famously, the Golden Proportion Ø,

where: $Ø = (\sqrt{5}+1)/2 = 1.618...$, and: $Ø = 1 + Ø^{-1}$.

III. The Value of the Hierarchy

In particular, I draw attention to the value of the hierarchy as an organizing principle [7], [8]. Under the influence of postmodernism, we have passed through a period of disenchantment with hierarchical structuring of complexity. But structural morphology emphasizes the near-universality and profound importance of this form as an organizing principle, to the point where it is considered that any significant level of complexity beyond the most simple requires for its organization some form of hierarchy [9]. Critical to this thinking is the qualitative proportioning of resources, so that resources are not distributed in a homogenous manner, but instead exhibit heterogeneous difference (here, quantitative differentiation can then emerge from the qualitative). This heterogeneity can then enable a system to achieve a more optimal state. This hierarchical principle is readily evident in Korean settlement patterns, recognizable in the concentration of population and resources in Seoul, Busan, then Daejeon, Daegu, through to smaller provincial centers.

I proceed to a straightforward partition of the field of enquiry, that provides a degree of mathematical treatment and hence abstraction, which allows for objectivity and enables intentional modification.

Consider the field of all tertiary EFL students in Korea. Further assume that in general, a higher level of existent English ability correlates with a higher aptitude for learning English, and that both these factors correlate with a greater motivation to engage in English learning. Conversely, assume that a lower level of English ability will tend to correlate with a lower aptitude for learning English, and with a lesser motivation and greater resistance to engage in English learning. Of course, individual differences mean these correlations will not always hold; however common sense suggests that they are likely to characterize typical Korean learners of English.

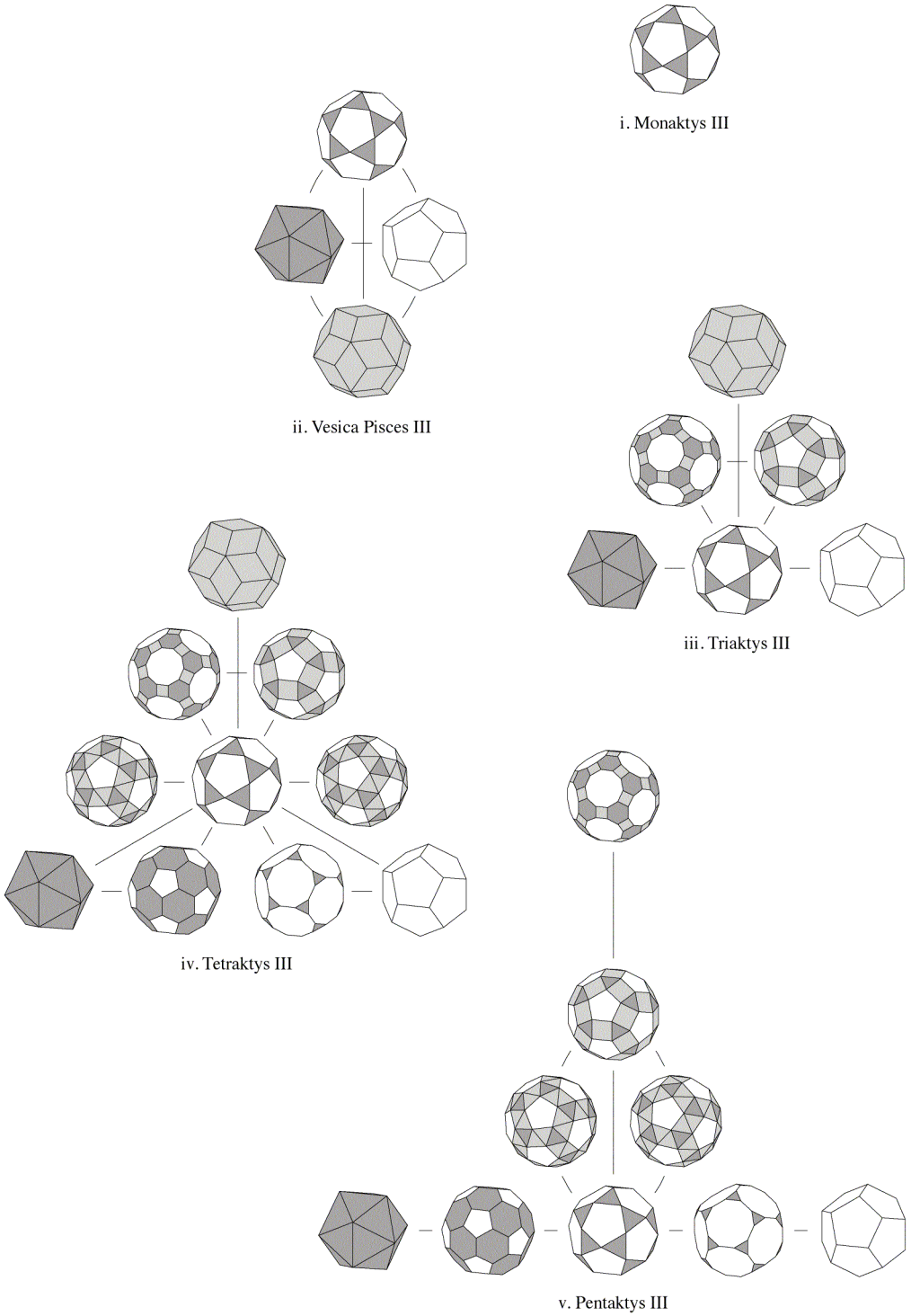

Fig. 1. Natural Harmony: The Unfolding of Inner Order in the Platonic and Archimedean Polyhedra - Class III of {2,3,5} Symmetry (author's research, see [5])

A simple partition of these students would then differentiate just three groups:

- High-level of English
  [high ability + high aptitude to learn + high motivation];
- Mid-level of English
  [mid ability + mid aptitude to learn + mid motivation]; and,
- Low-level of English
  [low ability + low aptitude to learn + low motivation].

Further assume (as seems reasonable) that there are more students of middle ability, aptitude and motivation than those of either high, or of low ability, aptitude and motivation, but not both high and low. Further assume (as experience and anecdotal evidence would suggest) that there are more students of low ability, aptitude and motivation than those of high ability, aptitude and motivation.

A convenient way to quantify these qualitative proportions is then by means of a Pythagorean 3:5:4 proportioning. An approximation of reality is therefore assumed that identifies 3/12 (a quarter) of students as being of high ability, aptitude and motivation; 5/12 (five/twelfths - nearly a half) of students as being of middle ability, aptitude and motivation; and 4/12 (a third) as being of low ability, aptitude and motivation:

3: High English
[high ability + high aptitude to learn + high motivation];

5: Middle English
[mid ability + mid aptitude to learn + mid motivation]; and,

4: Low English
[low ability + low aptitude to learn + low motivation].

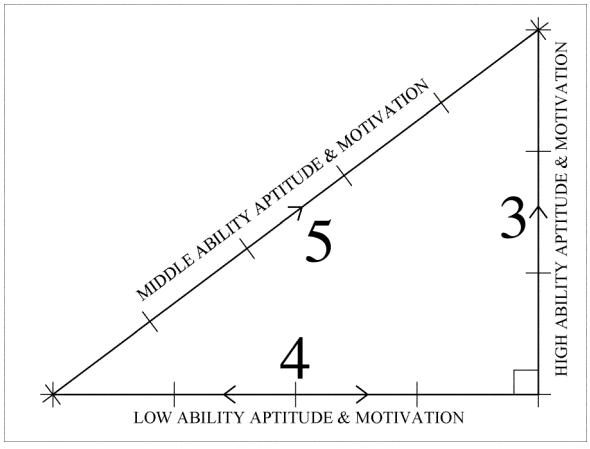

Fig. 2. 3:5:4 Pythagorean Triangle of Ability, Aptitude and Motivation

This is illustrated in a {3:5:4} Pythagorean Triangle [10], shown in Fig. 2 above. The vertical edge of 3 units represents those of high ability, aptitude and motivation, who may be imagined to soar directly towards communicative competence and fluency. The sloping edge of 5 units represents those of middle ability, aptitude and motivation who ascend more gradually towards such competency. The horizontal edge of 4 units represents those of low ability, aptitude and motivation, whose progress is more problematic.

I make three further assumptions: 1. that the resources available to teach these students English are finite and therefore limited; 2. that as presently distributed they are inadequate; and 3. that they are essentially constant, so that they are unlikely to substantially increase or decrease in the foreseeable future. These resources include native English-speaking instructors, their contact hours and the classrooms available, but also may be taken to include ancillary resources such as classroom computers, overhead projectors, etc.

IV. The Present Situation

At present, deployment of resources to these students is largely homogenous, so that the proportions of Students and Resources correspond:

TABLE 1
PRESENT HOMOGENOUS DEPLOYMENT OF RESOURCES

Ability/Aptitude /Motivation	Proportion of Students	Proportion of Resources
High	3	3
Mid	5	5
Low	4	4

I make a further assumption that appears quite reasonable: that those with higher ability, aptitude and motivation will, with the application of a reference quantity of resources, learn more English when compared with those of lower ability, aptitude and motivation exposed to the same reference quantity of resources. Thus educational achievement is assumed to be proportional to ability, aptitude and motivation. I then assign what I term AAM values of Educational Ability, Aptitude and Motivation on the basis of the squares of the first three integers, i.e. of 1 x 1 = 1 for Low, 2 x 2 = 4 for Middle, and 3 x 3 = 9 for High. This weighting (which though arbitrary, is considered, and derived from intuition) assumes, perhaps optimistically, that some positive outcome is derived from having students of low ability, aptitude and

motivation exposed to valuable educational resources, whereas in reality outcomes may even be negative, as poorly performing students become even more disillusioned and resentful. The weighting also assumes that students of higher ability, aptitude and motivation are not over-exposed to resources, and are not likely to produce negative reactions.

Further assume, as also appears quite reasonable, that for a given level of ability, aptitude and motivation, exposure to increased educational resources correlates with greater educational development, so that (within reason), educational achievement across the board is proportional to exposure to educational resources.

So for any individual, a first approximation of educational output can be measured as:

Individual Educational Output
= (Level of AAM)
x (Level of Educational Resources exposed to)

TABLE 2
FIRST APPROXIMATION OF INDIVIDUAL EDUCATIONAL OUTPUT

Ability/Aptitude Motivation	AAM Quotient	Resources Deployed	Educational Output
Single High-level Student	9	1	9
Single Mid-level Student	4	1	4
Single Low-level Student	1	1	1

This can then be generalized to the student population and society as a whole, so that educational output of the student body can be measured as:

Collective Educational Output
= (Level of Educational AAM)
x (Level of Educational Resources exposed to)
x (Population of Students)

TABLE 3
EDUCATIONAL OUTPUT OF STUDENT BODY

Ability/ Aptitude/ Motivation	AAM Quotient	Total Resources	Population Students	Educat-ional Output
High-level Student Body	9	3	3	81
Mid-level Student Body	4	5	5	100
Low-level Student Body	1	4	4	16
Total		12	12	**197**

So given a homogenous distribution of resources - under which each student receives essentially the same resources irrespective of their English ability, aptitude or motivation - the application of a constant quantity of resources results in a collective educational output of 197 arbitrary (reference) units.

V. Two Alternative Scenarios

The broad aim remains to raise the general level of student literacy in English and therefore the EFL literacy of the society as a whole. Instead of aiming at having each and every Korean student improve their English linguistic ability by the same amount - irrespective of ability, aptitude or motivation - instead give preferential treatment to those who are capable and who wish to learn. Valuable limited resources may be redistributed, by recognizing that at the lower levels of AAM, resources are being wasted, and at the higher levels, inadequate resources are being applied. Therefore, consider the situation in which students of low AAM are not required to learn English, firstly as an option, and secondly as a mandate.

A. Scenario 1

I further assume that these low-level students are given the choice of learning English if they wish to, while assuming that most will choose not to. This may be quantified by assuming that one-quarter of the low-level students elect to continue their English studies, i.e. 1/12 of the student population. This frees up 3/12 of the available resources for redeployment (which increases demand for non-English resources).

Further assume also that those 3/12 re-deployable resources are apportioned in the ratio 2:1, so that 2/12 extra resources are applied to the high-level AAM students, and 1/12 to the mid-level AAM students. This could be achieved by either increasing native English-speaking teacher contact hours by 2/3 for the high-level AAM students and by 1/5 for the mid-level AAM students; or by reducing class sizes accordingly, so that high-level AAM class sizes reduce to 3/5 of what they were previously, while mid-level AAM classes reduce to 5/6 of what they were previously. *In either case, the total number of contact hours worked by native English-speaking teachers remains the same; teachers are simply redeployed from lower level classes to higher*. So what was a class size of 25 high-level AAM students becomes just 15 students; what was a class size of 30 mid-level AAM students reduces to 25 students; and for low-level students, either class sizes reduce from say 24 to 6, or more likely, class sizes are held about the same but only one fourth of the classes are now required, while the remaining 3/12 students, 3/4 of the low-level AAM students, freed from the constraints of English study,

now study other subjects more suited to their temperaments, abilities, and interests (this requires the use of more affordable non-English resources in compensation).

Consider the educational output of this scenario:

TABLE 4
SCENARIO 1:
DEPLOYMENT OF RESOURCES WITH OPTIONAL LOW-LEVEL EFL

Ability/Aptitude/ Motivation	AAM Quotient	Total Resources	Population Students	Educational Output
High-level Students	9	5	3	135
Mid-level Students	4	6	5	120
Low-level Students A	1	1	1	1
Low-level Students B	(1)	0	3	0
Total		12	12	**256**

Under this redeployment, the total education output for tertiary Korean learners of English exposed to native English-speaking instructors has risen from 197 to 256, a percentage comparison of about 130%. The same resources have been applied, but more expeditiously; students of low AAM who do not wish to study English may concentrate on other matters.

B. Scenario 2

I now assume instead that all low-level AAM students are excused from studying English irrespective of whether they wish to continue to study it or not. I also assume that the freed 4/12 available resources are distributed so that the high-level AAM students are given an extra 3/12 resources, while the mid-level AAM students are given a 1/12 increase in resources as before. The gains now are even more impressive. This could be achieved either by doubling native English-speaking teacher contact hours for the high-level AAM students and as before by increasing contact hours by 1/5 for the mid-level AAM students. Or, class sizes are reduced accordingly, so that high-level AAM class sizes reduce to one half of what they were previously, while as before, mid-level AAM classes reduce to 5/6 of what they were previously. *In either case the total number of contact hours worked by native English-speaking teachers remains the same; teachers are simply redeployed from lower level classes to higher*. A class size of 30 high-level AAM students becomes just 15 students; a class size of 30 mid-level AAM students reduces to 25 students; and all low-level students, freed from the constraints of English study, now study other subjects to which they are better suited (though requiring more affordable non-English resources):

TABLE 5
SCENARIO 2:
DEPLOYMENT OF RESOURCES WITH EXCLUDED LOW-LEVEL EFL

Ability/ Aptitude/ Motivation	AAM Quotient	Total Resources	Population Students	Educat-ional Output
High-level Student Body	9	6	3	162
Mid-level Student Body	4	6	5	120
Low-level Student Body	(1)	0	(4)	0
Total		12	12	**282**

The total education output for tertiary Korean learners of English exposed to native English-speaking instructors then rises from a current 197 units to 282 units, a percentage comparison of over 143%.

TABLE 6
COMPARATIVE EDUCATIONAL OUTPUT

Situation/ Scenario	AAM Quota	Totals of Resources	Populations Students	Education Output	% Gain
Homogenous Deployment	9:4:1	3:5:4	3:5:4	**197**	**Base**
1: Low-level EFL Option	9:4:1	5:6:1	3:5:4	**256**	**30%**
2: Low-level EFL Exclude	9:4:1	6:6:0	3:5:4	**282**	**43%**

VI. Synergies Are To Be Expected

Further, synergistic effects are to be expected, as the structural morphological form of the penetration of English ability in Korean society moves from a relatively diluted distribution to approximate more clearly a well-defined hierarchical structure. The educational output of the high-level AAM students rises dramatically from 81 to 135 units in the first scenario (a 166.67% comparison), and even more dramatically doubles from 81 to 162 in the second scenario. And in both scenarios, the educational output of the mid-level AAMA students rises significantly from 100 to 120 units.

The effects of this differentially deployed optimization of educational output would likely permeate further throughout society. For any twelve representative students, three (one/quarter) will have the benefit of two/thirds more (first scenario), or twice (second scenario) the educational resources compared with the present situation. Five (five/twelfths) will have the benefit of one/fifth more benefits; while the

remaining four (one/third) will be freed from what they regard as unnecessary English study - but will instead have the valuable opportunity to study other fields that they regard with more interest, though they then require non-English resources.

VII. Conclusion

I present a speculative proposal that depends upon several assumptions. However, while the specific quantitative differentiation of variation employed may appear arbitrary, the qualitative analysis that underlies it is eminently reasonable. A degree of concentration of resources which favors those with relatively higher levels of L2 English ability, aptitude and motivation, at the expense of those with lower levels, is likely to significantly increase the overall educational output in society resulting from the deployment of the limited resources that are available. This illustrates the structural morphological advantages of the heterogeneity implicit in a hierarchical form, in comparison to a homogenous distribution of resources that is made irrespective of ability, aptitude or motivation.

By attending first to natural harmony (which I have illustrated through geometric and numerical examples) - rather than to political considerations - pedagogical resources that have been transmuted in the educational environment from applied resources to inner learning are more effectively deployed. The effects on the whole system will continue to yield benefit, as virtuous learning cycles are nurtured, become established, and in their fullness of time reach maturity.

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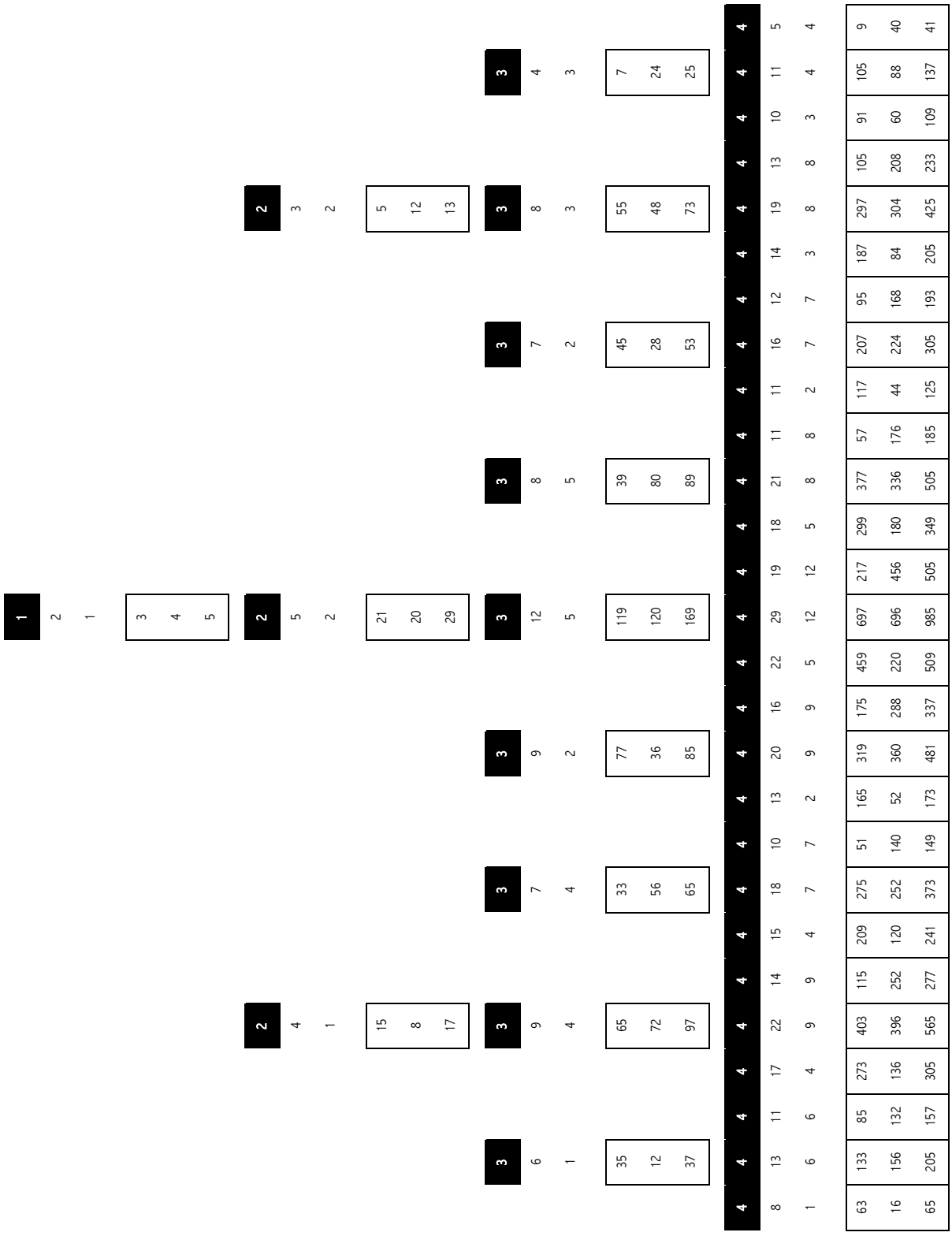

Fig. 3. Natural Harmony in Number and Space: First four generations of Pythagorean Triangles with half-angles (author's research [11] and after Knott [12])