Chapter XI

CIRCULAR SPACE TRUSSES ~ A CENTRALIZED STRUCTURAL GEOMETRY FOR THEIR DESIGN AND PREFABRICATION

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SYNOPSIS

The archetypal geometry of the center is given by the regular Star and its counterform the Polar Zonagon Mandala. By monoaxial development, a useful structural geometry for circular space trusses and domes is obtained of a web of equal length members in parallel sets that meet at modular angles of intersection. The projective geometrical nature of the mandala in its metrical expression develops zonahedral properties, which facilitate design and fabrication. Specific applications are indicated in Space, with dynamic structures which contract and expand about an axis. This paper condenses extensive research documented in the author's doctoral dissertation and book The Aesthetics of the Sacred.[1]

In this paper, and in much of the author's work in general, structural geometries are explored, and from them structural forms are then deduced. Geometries which structure space may be intuited and developed as pure geometrical entities, prior to their application as structural forms. A structural geometry is ideal, and the question of it being structurally stable or unstable is not relevant. For example, a cubic lattice has specific harmonic geometrical properties which may be evinced; these apply irrespective of the structural application in which that geometry may be exploited. Only when the geometric structure is physically concretized in a load-resisting application do issues of structural equilibrium and adequate strength become relevant. Those issues (of structural equilibrium) may then be achieved through triangulation, stiff connectors, or diaphragms etc.

The author's research has shown that it is beneficial to explore structural geometries fully as theoretical entities (for example in the computer-generated graphic environment). Specific structural applications then become apparent. The virtue of this approach is that the structural geometries respect and to a degree evince the natural structural qualities of space prior to their being compromised by the contingent factors of their physical realization. These geometries by virtue of their projective harmonic nature are very regular, effective and economic in enabling specific structural applications to be conceived and realized.

THE NATURAL GEOMETRY OF THE CENTER

The center is the author suggests the most archetypal spatial form. Mircea Eliade shows the center to be associated with the Sacred; a cosmos comes into being from its center, and it is reabsorbed back through the center at the end of its natural life. Spatially, we encounter two- and three-dimensional centers. The geometry of the three-dimensional center is either approached on a polyaxial or monoaxial basis. The regular polyaxial center is approached through the symmetries of the regular and semi-regular polyhedra,[2] and their development through configurations such as the centralized zonahedral complexes the author elsewhere develops.[1] Alternatively the monoaxial geometry is developed from the geometry of the two-dimensional center, which is the main concern of this paper. It therefore makes sense to address the natural geometry of the two-dimensional center (in the plane), and from that develop natural geometries of the monoaxial three-dimensional center. From these various geometrical expressions, structural applications are deduced.

Space structures and trusses are often developed as two-dimensional planar or curved surface forms (as in the R. Buckminster Fuller designed geodesic dome of the World's Fair U.S. Pavilion). Although they have depth, they are essentially surface forms. Further, structures are generally of finite extent. Where these are of planar form, their most fundamental expression is circular (just as in microgravity in three-dimensions the archetypal spatial form of finite structures is spherical). Accordingly where finite planar space structures are indicated, the natural structural geometry of the two-dimensional center proves appropriate. Monoaxial developments into three-dimensions are particularly relevant to applications on earth where a gravitational field is encountered which is to all intents and purposes monoaxial, except in structures of very great extent. It is then sensible to align the axis of the structural geometry to that of gravity, as domical forms testify. But such developments are also applicable in Space operations in monodirectional uses such as antennae or telescopes, or where surface structures are required. In particular they are suited to dynamic space structures which expand and contract about an axis.

In the two-dimensional geometry of the center in the plane, the archetypal geometry is given by the regular polygons, more specifically by their development into the infinite regular stars on the one hand, and by their finite counterforms, which the author terms the polar zonagon mandalas. It proves useful to restrict the frequencies of these firstly to even frequencies (as odd frequencies are not as symmetrical); secondly for the purposes of illustration of this paper to a frequency of twelve. But their application is envisaged at whatever frequencies are appropriate to the specific use for which they are intended.

The regular Star and its counterform the Polar Zonagon Mandala are metrical expressions of archetypal projective geometrical configurations. Through their projective nature, their elements such as nodes, line segments, polygons and circles firstly exhibit important projective properties such as coincidence, colinearity, and what is termed coconicity (i.e. lying on the same conic). Secondly their relative proportions, angular and linear, are harmonic. When these projective forms are expressed in their most regular metrical form, they exhibit metrical properties of equal-length, parallelism, and modularity of angle. Usually these properties are maintained when the geometries are developed into three-dimensions. Harmonic properties are elegantly shown therefore in structural geometries with members of equal length which occur in parallel bundles, and meet at modular angles of intersection, as characteristic of zonahedral geometries.

Usually such metrical harmonic geometries generate either equal measure of length, with harmonic angular measure; or equal measure of angle, with harmonic linear measure. The significance of the centralized geometry presented in this paper is that while it is fundamentally an expression of equal measure of angle, it also conveniently develops equal measure of length. Consequently it is very useful for centralized applications.

Accordingly these structural geometries exhibit properties which enable them to be readily conceived, reconfigured in the mind's eye during design, proportioned, modelled in the computer-generated graphic environment, and fabricated as specific structures. Transformations are possible, through circumferential rotation, or radial expansion and contraction, about an axis. These suggest specific applications in Space operations as dynamic structures which can be deployed in various states of openness and closure; or which fold and unfold, and could be delivered in a contracted state before being deployed.

THE REGULAR POLAR ZONAGON MANDALA

This form can be visualized and generated in a variety of ways (refer Fig. 1.i-iv):

- From an initial burst of vectors from the center, adjacent vectors mate to produce parallelograms, and the sequence expands outwards, meeting a natural limit to the expansion which is a two-frequency regular polygon. It then reflects from the boundary, contracts, and eventually implodes and is reabsorbed through the center.
- The extended regular star is decomposed into concentric star rings as line segments zigzag out and in; these are resized and their order reversed to become mandala rings.
- If a polar inversion of the extended regular star is made about its in-circle, each line inverts into a circle. The polar zonagon mandala is then configured as an eccentric overlapping circular set of circles of equal radius, equally spaced about and centered on a central reference circle, each coincident at the center, their intersections giving the nodes of the mandala. This regular polar inversion is why the author regards the mandala as the counterform of the star. Straight line-segments then connect adjacent nodes.
- Thus the mandala can be reconfigured as an eccentric overlapping set of regular polygons centered on the nodes of a central regular polygon, each sharing a central node.
- The mandala can be seen as a checkerboard "hit-and-miss" pattern connecting nodes of intersection of a central ray of equispaced lines and coincident circles.

Structural Application of the Planar Mandala:

If the outer mandala ring is deleted, the mandala can be constructed as a tensile lattice, fashioned by curving an ordinary tensile net of square mesh. The central ties are doubled. The curvature in Fig. 1.v of the square net of pin-jointed struts into the circular mandala, forms a structural geometrical transformation of circumferential rotational closure for folding circular trusses and domes.

MONOAXIAL DEVELOPMENT OF THE MANDALA

The Dome Strand:
If from the plane, line-segments are inclined as vectors of equal slope, a dome formation is generated as in Figs 1.i, 2.iii & 3.ii. Parallel bundles of angled struts are generated about the structure. Individual eccentric polygons develop into regular eccentric polygonal helices. For the case of infinite frequency, the surface of curvature is the surface of rotation of the sine-wave. Nodes of the finite case lie on that surface, and the eccentric circles of the mandala transform into regular eccentric helices lying always on that surface, passing through both center and periphery. This is an elegant behavior. The dome structure suggests various structural applications; and the geometry appears to characterize the structural and surface decorative geometry of the Islamic dome.[3]

A Simple Centralized Space Truss:
The polygonal Mandala is axially displaced and duplicated with corresponding points connected with verticals to give a *linee occulte* grid for a single layer centralized space truss shown in Fig. 2.i. The vertical displacement is made equal to the edge length of the Mandala, hence the vertical faces of the truss are square. (The structure is also formed by curving a single layer of a cubic lattice in the horizontal plane as in Fig. 1.v). Stiff connectors, triangulators, or diaphragms ensure structural stability. If replicated axially, a multi-layer prismatic lattice is obtained.

Wedding Cake Dome:
In the simple space truss, mandala rings have become star prisms, with square sides folding in and out about their periphery. These recombine in the wedding cake dome of Fig. 2.ii, from which various structural domes are deduced.

Octet Truss Dome:
For example, using pin joints, transverse elements with surface diamond triangulators are added, to provide an octet truss dome shown in Fig. 2.v composed of component octahedra and tetrahedra - the domical transformation of the octet truss. Transverse triangulators between the two skins and horizontal surface diamond triangulators lie in the common horizontal planes of points of the domes.

Single-Layer Space Trusses are Generated, which can Contract and Expand:
Individual eccentric polygons of the mandala develop into eccentric regular polygonal prisms of the simple space truss. In Fig. 2.vii diagonals of square side-faces of the prisms connect in either of two ways to generate skew polygons. These combine to generate the webs of single-layer space trusses of either center-up or center-down orientation of Figs 2.viii-ix & 3.i. Each web expands or contracts regularly about its axis, as in Figs 4.i & ii, as the angle to the horizontal of its members increases on compression or decreases on expansion. At any stage of expansion the structural geometry is such that web members all have the same degree of slope whilst retaining in plan their orientation.

These Combine in the Double-Layer Space Truss, which can Contract and Expand:
Both kinds combine in Fig. 2.x to generate the double-layer space truss of Fig. 3.iv-v. Paired diagonals of a square face function as scissor elements which allow the truss to regularly compress and expand transversely about its axis, which is shown in Fig. 4.iii.

Dome Sheaths:
Figs 3.iii & v show trusses axially combined into single and double density sheaths, which can contract and expand. They contain dome strands within them.

Folding Vertical Displaced Dome:
Scissor elements of two different lengths, given by the diagonals of the parallelograms of Fig. 2.iv, generate the folding dome of Fig. 2.vi which contracts and expands about its axis. If of low aspect, it suggests a Space antenna.

Folded Plate Configurations:
Folded plate constructions shown in Fig. 5 are derived from the centralized space truss geometry to generate centralized octet trusses for roofs.

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1 Meurant, R.C., *The Aesthetics of the Sacred - A Harmonic Geometry and Philosophy of Sacred Architecture (Third Ed.).* The Opoutere Press, Auckland, 1989, pp. 5-35 and color plates A.1-E.5.

2 Meurant, R.C., *A New Order in Space - Platonic and Archimedean Polyhedra and Tilings.* International Journal of Space Structures, Vol.6 No.1, University of Surrey, 1991.

3 Meurant, R.C., *The Integral Space Habitation ~ Towards an Architecture of Space (First Ed.).* The Opoutere Press, Auckland, 1989; and Meurant, R.C., *Structure, Form, and Meaning in Micro-Gravity ~ The Integral Space Habitation.* International Journal of Space Structures, Vol.5 No.2, University of Surrey, 1990.

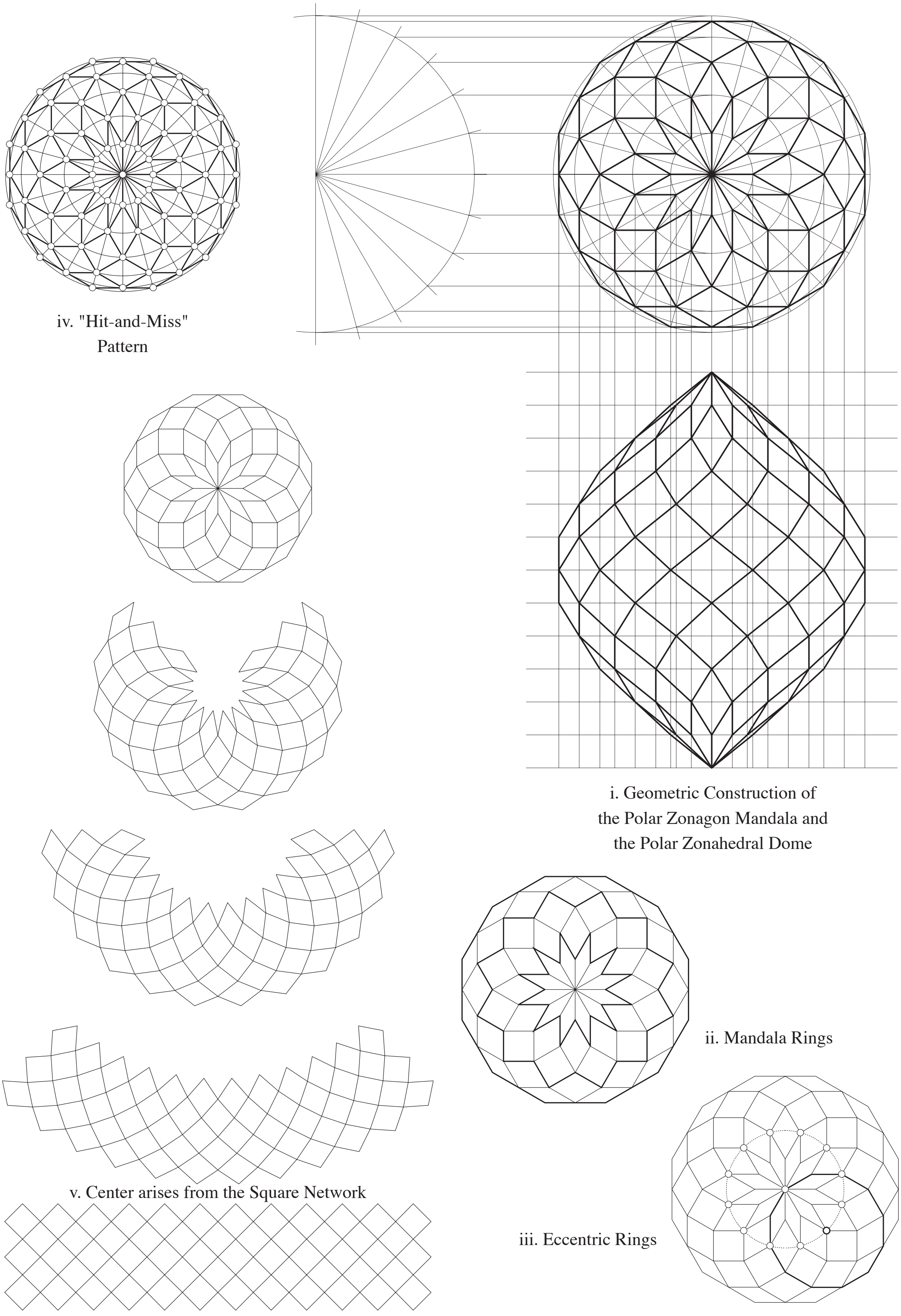

Figure 1 : Construction and Properties of the Polar Zonagon Mandala

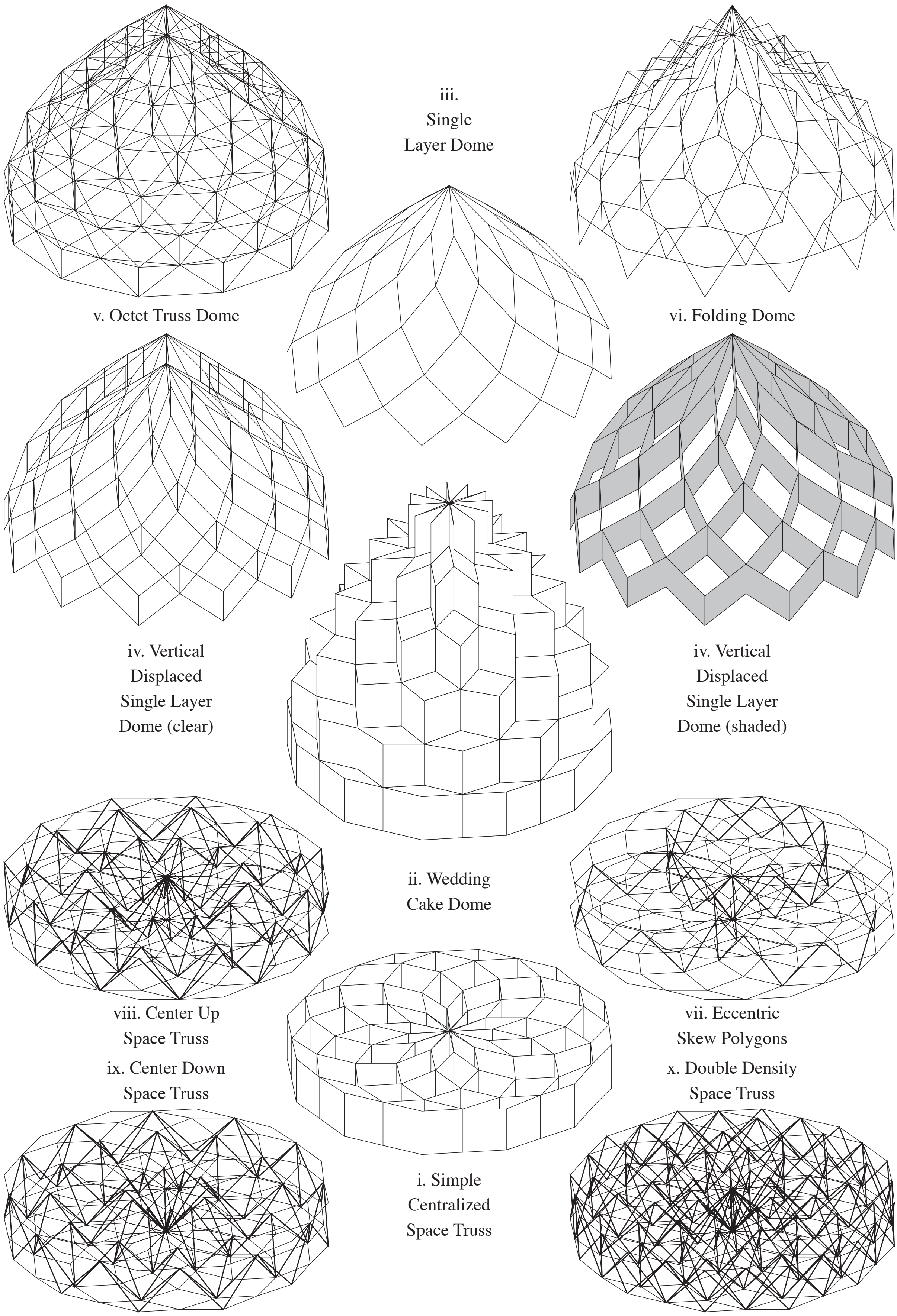

Figure 2 : Monoaxial Development of the Mandala into Trusses and Domes

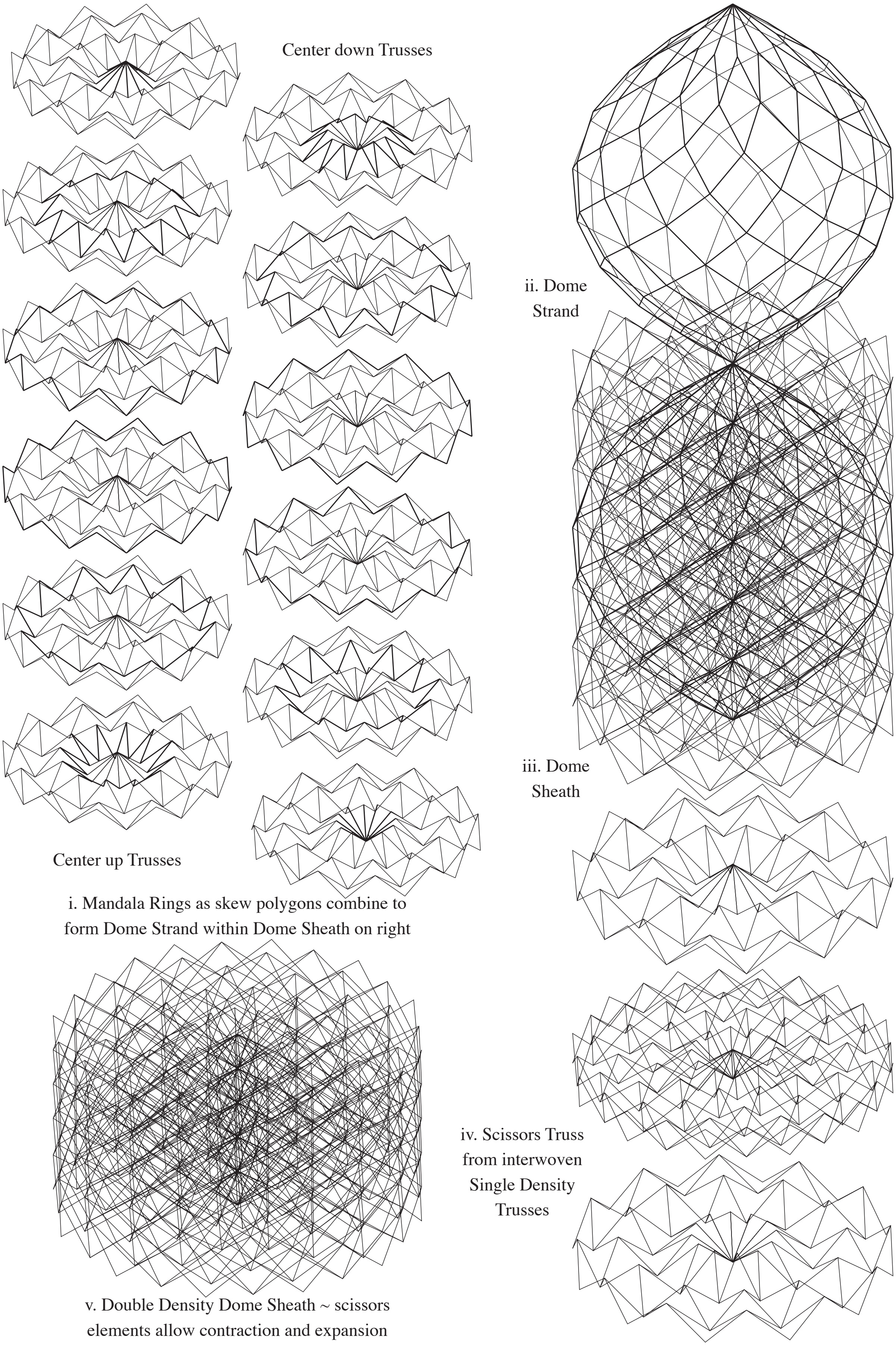

Figure 3 : Single Density (above) and Double Density (below) Trusses and Dome Sheaths

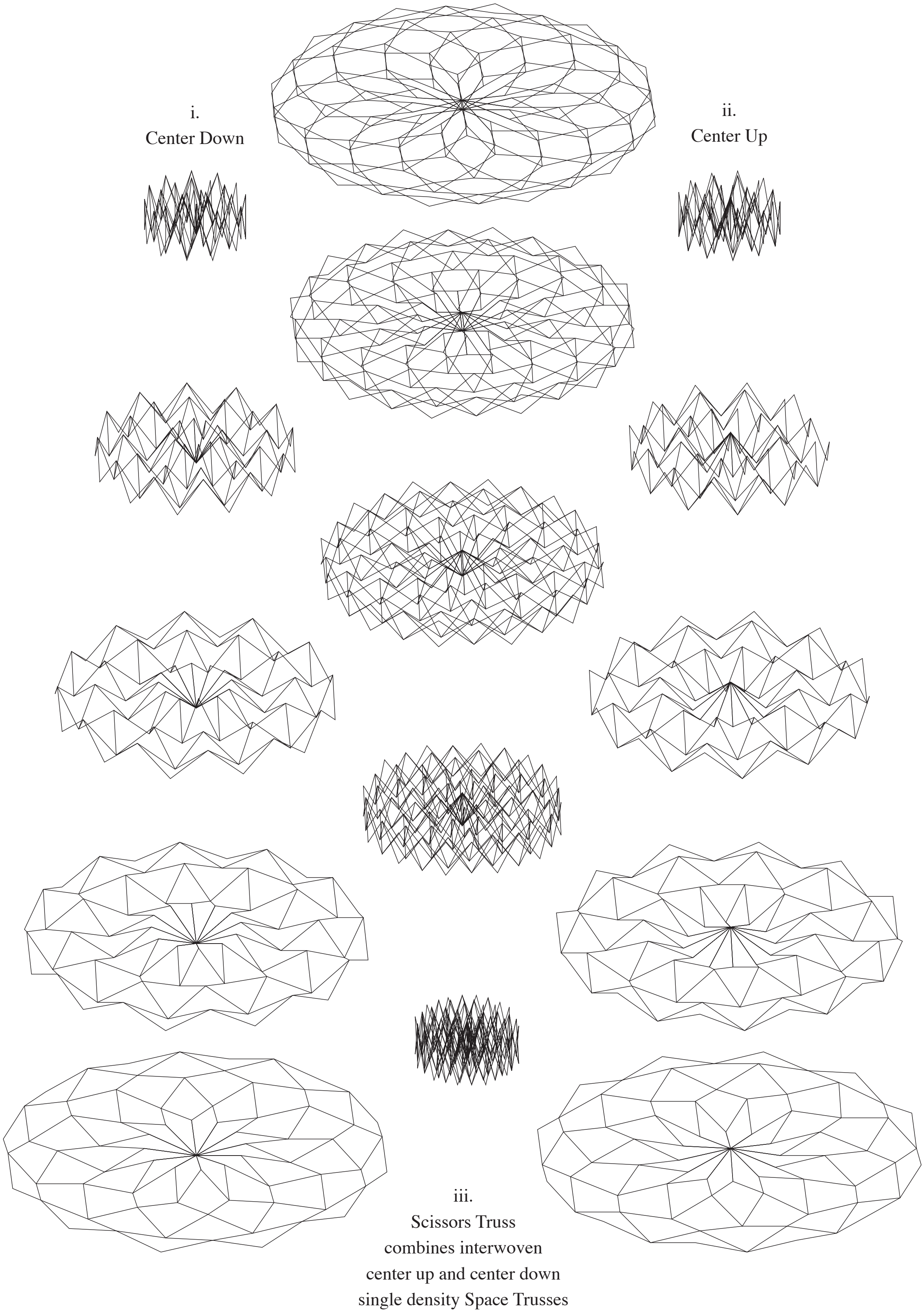

Figure 4 : Expansion and Contraction of Single Density (outside) and Double Density (inside) Folding Centralized Space Trusses

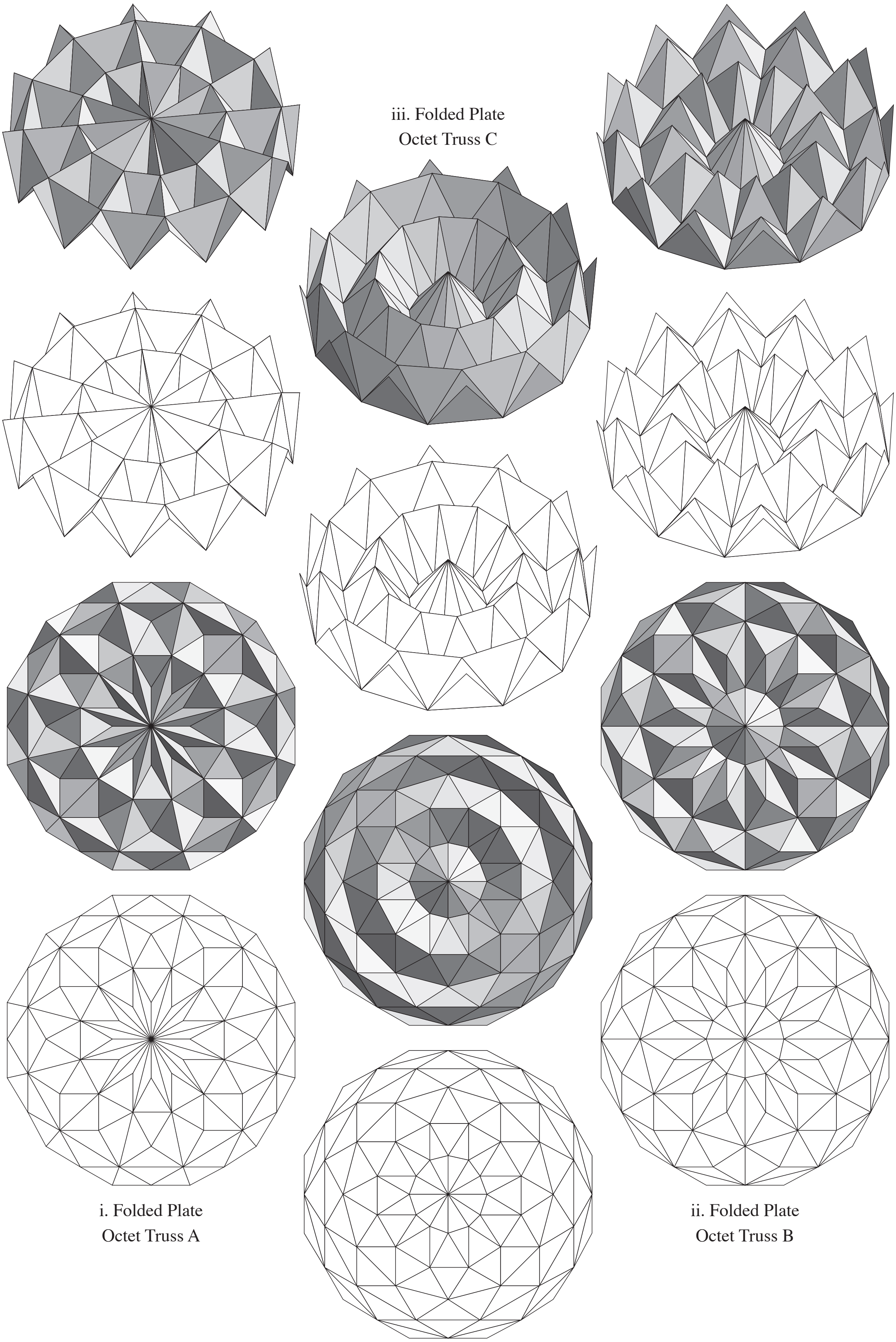

Figure 5 : Some Folded Plate configurations based on the Single Density Space Truss