

Chapter X

THE TETRAKTYS OF POLYHEDRA ~ THEIR HARMONIC STRUCTURE ACCORDING TO TRADITIONAL GEOMETRIC SCHEMA

FIRST PRESENTED TO THE FOURTH INTERNATIONAL CONFERENCE ON SPACE STRUCTURES,
UNIVERSITY OF SURREY, GUILDFORD, UNITED KINGDOM, SEPTEMBER 1993
AND PUBLISHED IN THE CONFERENCE PROCEEDINGS.

SYNOPSIS

In a previous paper "A New Order in Space" the author advanced a threefold order for the regular and semi-regular polyhedra. He now revises that order in the light of certain traditional schema of Sacred Geometry, particularly the Vesica Pisces and the Pythagorean Tetraktys. This situates Polyhedral Geometry within a

traditional perspective, and allows the dimension of human meaning to be incorporated. The polyhedra exhibit structural patterns which in a fundamental sense embody the qualities of space, and reveal the structure of the human psyche. Polyhedra, space and consciousness are perhaps different aspects of the same underlying reality, which may be termed Natural Harmony.

PREVIOUSLY THIS AUTHOR ADVANCED A THREEFOLD order for the regular and semi-regular polyhedra¹ with each class containing polyhedra which exhibit the same pattern of symmetry. Each pattern is characterised by neutral, male and female axes of symmetry with characteristic orientations and frequencies. The three classes are Class I: {2,3,3}; Class II: {2,3,4}; and Class III: {2,3,5}. Faces are neutral, male or female as they occur perpendicular to axes, except for paired neutral faces of the snub polyhedra. Within each class, polyhedral elements are disposed relative to one another according to a structure, and situated along sequences of truncation, rotational displacement, and transcendence.

Certain polyhedra reoccur according to the different symmetry patterns they exhibit, having regard to their coloring of faces. Individual polyhedral elements correspond precisely from class to class according to their position within the structure, and the sequences within each class correspond precisely from class to class.

The order of polyhedra is also extended to two further classes to account for various colorings of all but one of the regular and semi-regular tilings of the plane (the exception (3³,4²) being a degenerate case). The extra tiling classes are Class IV: {2,3,6} and Class V: {2,4,4}.

This paper revises the interrelationship of polyhedra into a more integral order, according to ancient schema of the traditional science of Sacred Geometry, in particular the Vesica Pisces and the Pythagorean Tetraktys. This situates the discipline of Polyhedral Geometry within a traditional perspective, which facilitates further research whilst incorporating the important dimension of human meaning.

Polyhedra exhibit structural patterns of interrelationship which embody fundamental qualities of space and reveal a deep structure of the human psyche which entertains them. This deep structure is not imposed, but rather discovered. Polyhedra, space, and human consciousness have embedded within them the potential for these orders of harmony. They are different aspects of the same underlying reality that may be termed Natural Harmony.²

SCHEMA OF SACRED GEOMETRY

The schema of Sacred Geometry which are relevant are the Circle and Sphere; the Vesica Pisces; the Trinity; and the Pythagorean Tetraktys. The Tetraktys is then developed into triadic arrays of other frequencies, with the Circle, Vesica Pisces, and a further schema the Star of David being considered as subsets or extensions of these.

Firstly the Circle is the most fundamental form of Sacred Geometry, and traditionally denotes the sacred center, the cycle of creation, maintenance and completion, and the perfection of totality. It exhibits symbolism adequate to the monad (identity), duad (sacred temenos and profane without) and triad (spiritual center, psychic domain, and physical boundary). The regular and semi-regular polyhedra are all bound by regular polygonal faces, each inscribable on a circle. In traditional symbolism these represent different qualities of manifestation. These faces are restricted to just the triangle, square, pentagon, hexagon, octagon, and decagon; and if the tilings are also included the dodecagon. Each face is disposed according to the frequency of axis of symmetry on which it lies, being either of single or double frequency. They are each constructed with compass and straight-edge, and embody $\sqrt{2}$, $\sqrt{3}$ or $\emptyset = 1.618...$ (and thus $\sqrt{5}$) proportional harmonies, which in tradition accord with qualities of generation, formation and regeneration.

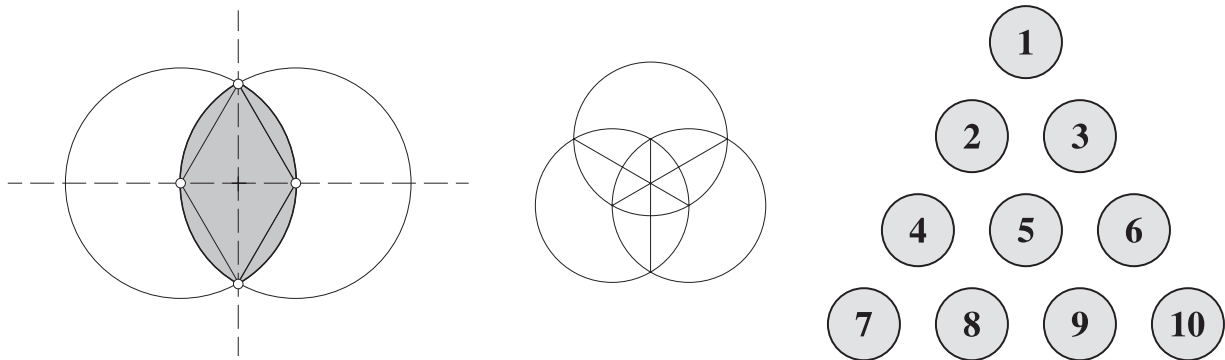


Figure 1 : The Vesica Pisces (Mother of Form), Trinity logo and the Pythagorean Tetraktys

The traditional symbolism of the Sphere is similar to that of the Circle, particularly as an exemplar of perfect form: a finite embodiment of the infinite.³ Critchlow shows each of the regular and semi-regular polyhedra constructed on the Sphere;⁴ the various polyhedra thus express different specific qualities of universal manifestation.

Secondly the Vesica Pisces, the most important schema of the Sacred Geometry of mediæval Christianity, is regarded as the “Mother of Form”. It represents an archetypal division of unity into duality, which generates a higher trinity. It also represents a quaternary, as the initial duality of centers generates a second duality of intersections; it thus organises the world into an above and below, and a left and right. This symbolism is later shown to be particularly rich and adequate to its use with the polyhedra. Its proportional harmonies are $\sqrt{3}$; but it is also used to form $\sqrt{2}$ and \emptyset harmonies.

Thirdly, a symbol of the Trinity is developed from three overlapping circles of equal radius, the center of each circle lying at the overlap of the other two circles to yield three interpenetrating vesica pisces. The three-in-one symbolises the integral interwoven threefold order of polyhedra, and hence represents the potential of three-dimensional space.

Fourthly, the Pythagorean Tetraktys, a four-frequency triadic array, exhibits symbolism of the central monad, the mirror-image duad, the triangular triad, the four level tetrad, and finally the decad as $1 + 2 + 3 + 4 = 10$. The integration of three- and fourfold symbolism is particularly apt, three symbolising the heavenly realm and four the earthly (also, $3 \times 4 = 12$, the Sun, and $3 + 4 = 7$, the Moon). The tetraktys was an essential part of the Quadrivium which, according to cabbalistic tradition, was preserved by inscriptions on the pillars of Seth when God destroyed the world by flood and flame. Proclus says:

- Sacred Number springs

*From th'uncorrupted Monad, and proceeds
To the Divine Tetractys, she who breeds
All; and assigns the proper bounds to all,
Whom we the pure immortal Decad call.*⁵

The triadic array is generalised to obtain a cosmogonic sequence of monaktys, duaktys, triaktys, tetraktys and pentaktys. The heptaktys is used with a subset yielding a further schema of traditional geometry the Star of David; or it is developed as an extension of the tetraktys. The Star of David usually represents the duality of complementary triads, but here the symbolism is of a threefold interwoven order of quaternary Vesica Pisces.

COSMOGONIC SEQUENCE OF THREE INTERWOVEN CLASSES OF POLYHEDRA

Firstly one may commence with the monaktys as the three-in-one symbol of the Trinity, which symbolises the interwoven harmony of the threefold order of the polyhedra. This also represents what Lawlor describes as the Hindu concentric cosmic model of polyhedra inscribed one inside another,⁶ one octave comprising Icosahedron, Dodecahedron, Cube, Positive and Negative Tetrahedra, and Octahedron through to inner Icosahedron.

Secondly the duaktys comprises three quasi-regulars, each the key solid of its class: the Class I Octahedron, the Class II Cuboctahedron, and the Class III Icosidodecahedron.

Thirdly, two forms of the triaktys are derived; firstly, each quasi-regular solid is located opposite its dual. Secondly the same dualities locate the respective male and female polar solids of each class, with male elements without and female within.

Fourthly the tetraktys locates three triplets at this level of manifestation, one for each class, of outward quasi-regular and withdrawn male and female poles to either side, the three triplets being symmetrically arranged about the central three-in-one. Quasi-regulars form an outer triangle; a hexagon consists of alternating male and female poles, which comprise two male and female interpenetrating triangles, with the polar opposites of each class lying opposite each other. (Here the tetraktys is shown within the Star of David).

THE STAR OF DAVID GIVES RISE TO THREE CLASSES OF VESICA PISCES

The tetraktys is developed into the Star of David by the addition of peripheral dual rhombic solids, being the duals of the quasi-regulars. Three interpenetrating Vesica Pisces one for each class surround a central three-in-one symbol. There are two outer triangles, one of the quasi-regular solids, the other of their duals, with each quasi-regular lying opposite its dual. There are two interpenetrating inner triangles, one of male poles and the other of female, with each male pole lying opposite its dual female pole.

This structure is teased apart into three separate Vesica Pisces, one for each Class. Each Vesica Pisces is reoriented into quasi-regular solid above and its dual below, with the male pole on the left and its dual the female pole on the right. The perfect quasi-regular above separates into the two dual polar regulars, and recombines in the dual solid below.

THE GRADUAL UNFOLDING OF INNER ORDER WITHIN EACH CLASS

A cosmogonic sequence for each Class represents the staged unfolding of inner order, from the single quasi-regular solid as source through to the full development into the complete class of regular and semi-regular elements. The duals of the quasi-regulars (which are not in general semi-regular) are at times included and at others excluded.

The sequence commences with the seed monaktys of the quasi-regular solid, which contains in potential the entire class. It proceeds to the quaternary Vesica Pisces of orthogonal dualities, with vertical axis of quasi-regular and its dual, and horizontal axis of dual polar extremes of the regular solids. The author elsewhere concludes that the regular solids though perfect, are really extremes polar to their mediating and harmonising perfect quasi-regular solid, which is thus a more perfect form. (However it might then be argued that such a solid, by virtue of being merely one element of a sequence of quasi-regulars, is therefore less than perfect). Its shadow below is its dual.⁷

The triaktys for each class locates the quasi-regular at the center of the base between its regular polar extremes on either side. The dual rhombic solid of the quasi-regular lies at the apex, so that quasi-regular and dual lie on the central vertical axis. The great and small rhombic element lie on either side on the second level, so that the neutral elements of first, second and third degree form the inner triangle.

This is developed into the tetraktys which comprises all of the elements of that class. In accord with its central nature, the quasi-regular element is located at the center, with its dual at the apex. Regular polar elements are located at two opposite vertices of the outer triangle. Their truncates are situated between them, along the base line. Great and small rhombic elements are located as before relative to the central quasi-regular element, forming a triangle which allows for the alternate sequences possible connecting the three. Enantiomorphs of the snub polyhedra lie either side of the central quasi-regular element.

Finally the subset of the pentaktys structures the completely unfolded order for each class (leaving aside the dual of the quasi-regular). The base line comprises the truncation sequence of regular male pole, truncated male pole, quasi-regular neutral center, female truncated pole and regular female pole. The rotation-displacement sequences arrange an inner Vesica Pisces of small rhombic Second Order element and quasi-regular First Order neutral center above and below, and intermediary left- and right-handed enantiomorphs of the snub polyhedra on the left and right. The central vertical axis erects a transcendent sequence of quasi-regular First Order element below, small rhombic Second Order element in between, and culminates in the great rhombic Third Order element on high.

Contemplation of this polyhedral order according to the Vesica Pisces and Pythagorean Tetraktys recalls Plato's dictum, which teaches that they mediate the world of the Good, the Beautiful and the True.

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- 2 Meurant, R.C., *Natural Harmony ~ A Structural Morphology of Consciousness?* 1^o International Seminar on Structural Morphology, Montpellier France, Sept. 1992.
- 3 Critchlow, K., *The Soul as Sphere and Androgyne*. Golgonooza Press, Ipswich.
- 4 Critchlow, K., *Order in Space*. Thames and Hudson, London, 1969.

- 5 Heninger, S.K. Jr., *Touches of Sweet Harmony - Pythagorean Cosmology and Renaissance Poetics*, The Huntington Library, San Marino California, 1974, p. 84 (quote translated by Thomas Stanley).
- 6 Lawlor, R., *Sacred Geometry - Philosophy and Practice*. Thames and Hudson, London, 1982, pp. 98-103.
- 7 Meurant, R.C., *The Myth of Perfection of the Platonic Solids*. Proceedings of the PAPER Conference on Myth Architecture History Writing, Auckland, 1991.

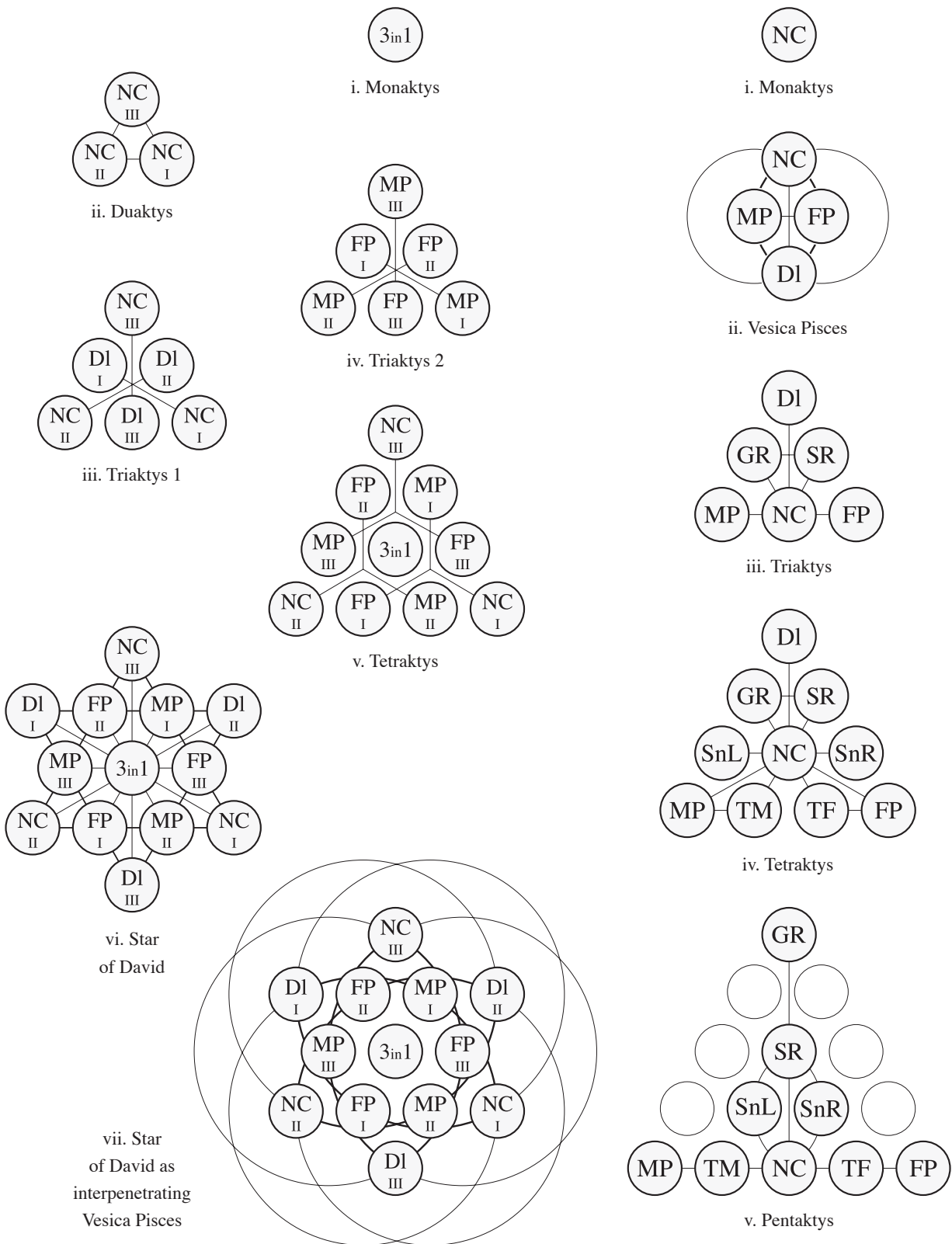


Figure 2 : Key to Schema of All Three Classes of Polyhedra

Quasi-regular Neutral Centre	NC
Great Rhombic Solid	GR
Left-handed Snub Solid	SnL
Truncate Male Pole	TM
Male Pole	MP

Figure 3 : Key to Schema of Individual Polyhedral Classes

DI	Dual Rhombic Solid
SR	Small Rhombic Solid
SnR	Right-handed Snub Solid
TF	Truncated Female Pole
FP	Regular Female Pole

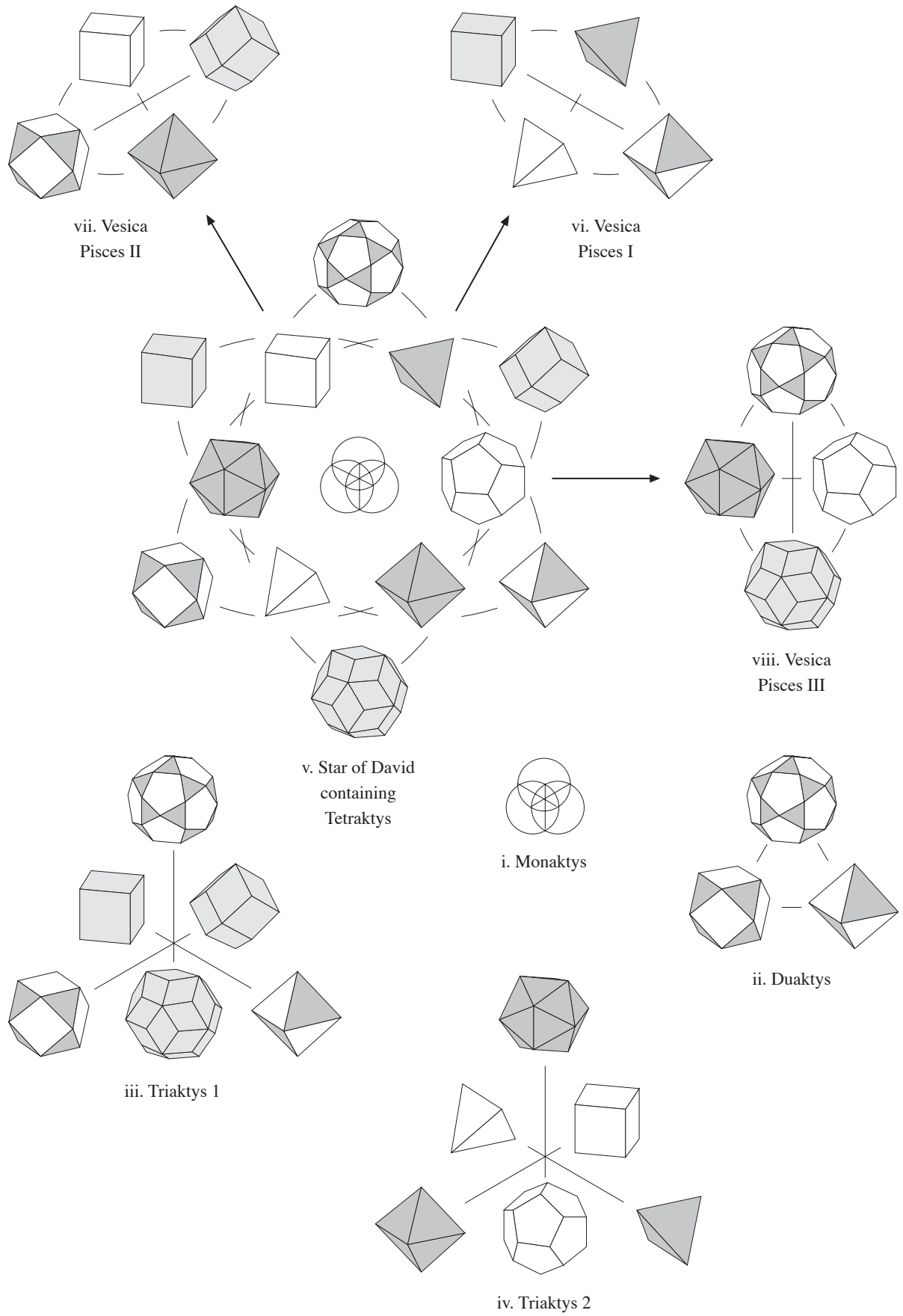


Figure 4 : Cosmogonic Sequence of All Three Classes of Polyhedra;
 Star of David (above) gives rise to three separate Vesicas Pisces.

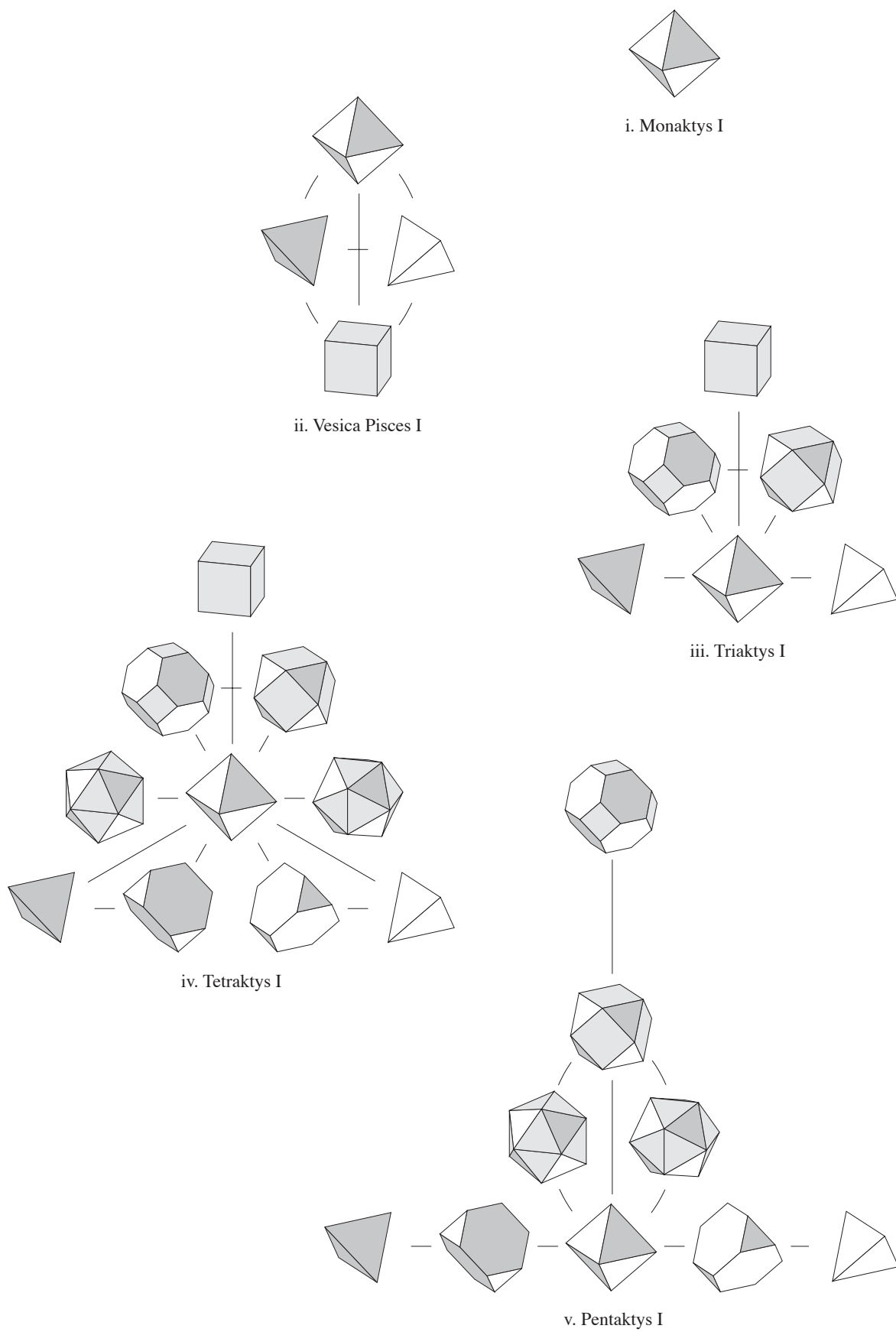


Figure 5 : The Unfolding of Inner Order ~ Class I : {2,3,3}

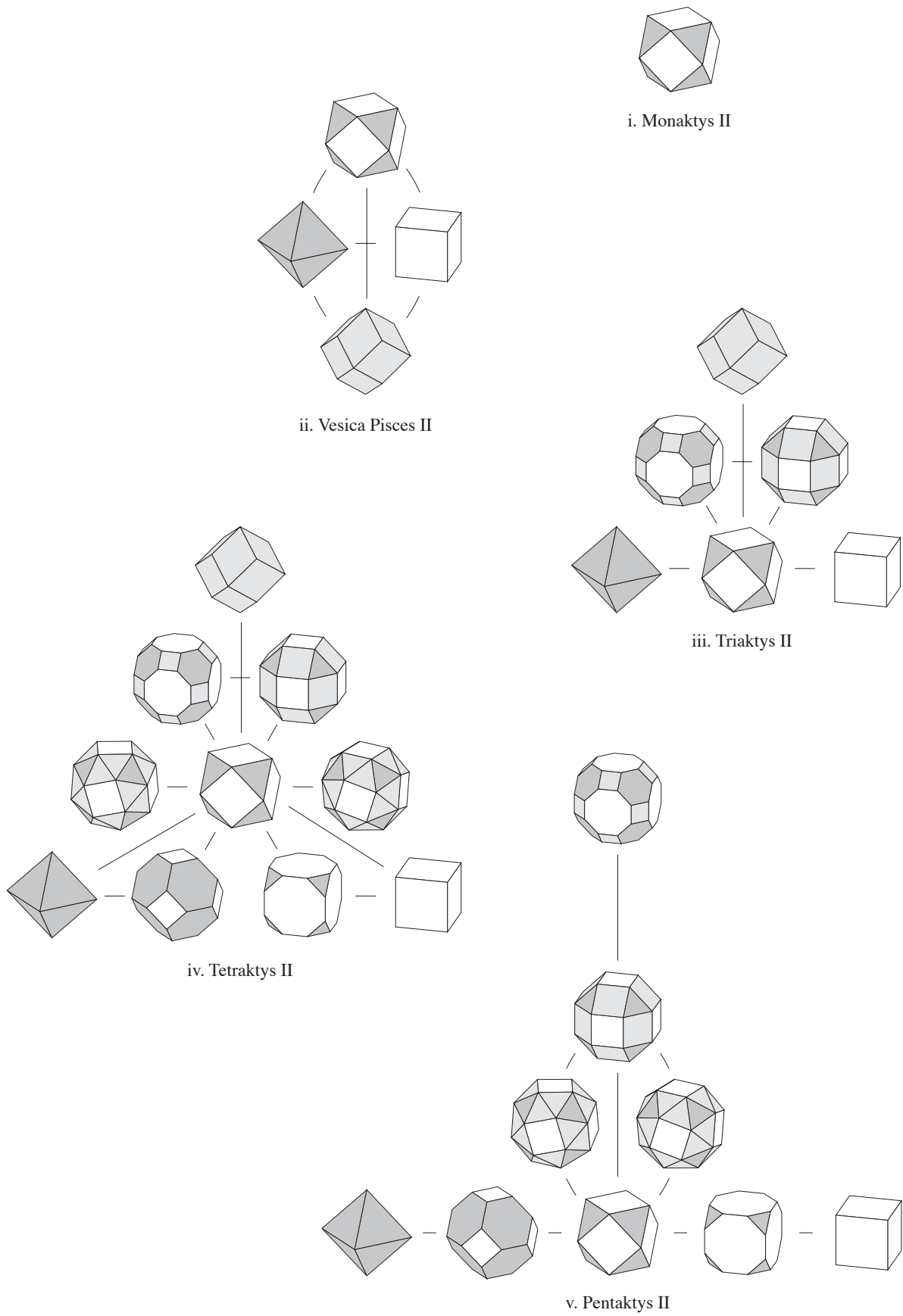


Figure 6 : The Unfolding of Inner Order ~ Class II : {2,3,4}

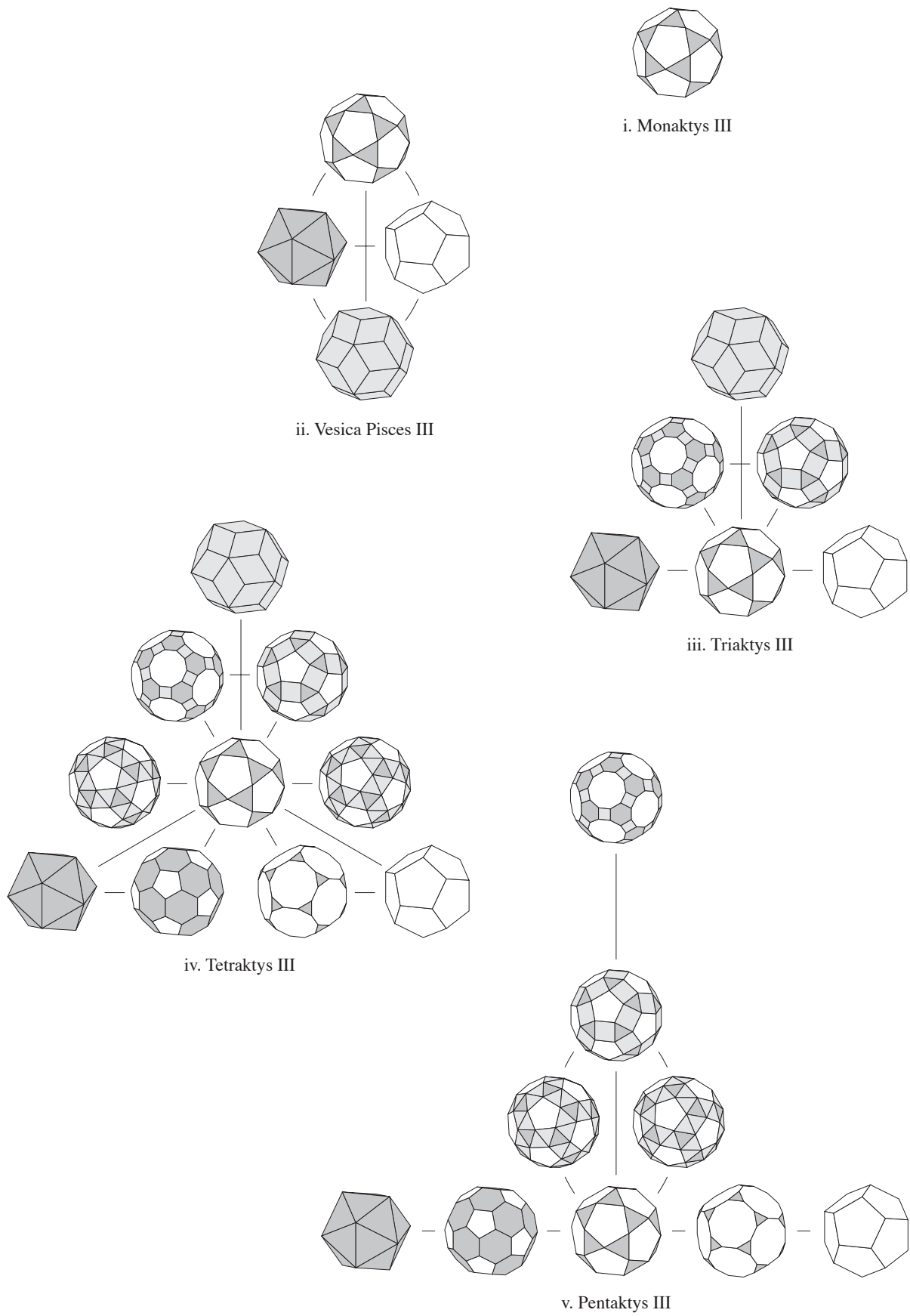


Figure 7 : The Unfolding of Inner Order ~ Class III : {2,3,5}