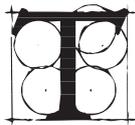


Chapter VII

THE MYTH OF PERFECTION OF THE PLATONIC SOLIDS

FIRST PRESENTED TO THE PEOPLE AND PHYSICAL ENVIRONMENT RESEARCH PAPER CONFERENCE ON MYTH ARCHITECTURE HISTORY WRITING, AT THE UNIVERSITY OF AUCKLAND, NEW ZEALAND, JULY 1991, AND PUBLISHED IN THE PAPER CONFERENCE PROCEEDINGS.



THE REGULAR AND SEMI-REGULAR POLYHEDRA HAVE exercised a deep fascination in the mind of man since their discovery in prehistoric times. More than a millennium before the time of Pythagoras or Plato, the Neolithic people of the British Isles were carving stone balls in designs that exploited the polyaxial symmetries of the regular and quasi-regular polyhedra and their duals. Keith Critchlow considers these may have had calendrical or astronomical significance,¹ and they are also thought by Aubrey Burl to have been fertility objects representing testes, being often placed in graves in pairs in association with phallic objects.²

The five regular solids known as the Platonic solids were so called not because they were discovered by Plato, but because of the special emphasis he placed upon them in expounding the Pythagorean cosmology in the *Timaeus*. There are no known prior written records referring to these solids; Plato tells us in the *Republic* that stereometry had not been adequately investigated at the time the dialogue is supposed to take place (*Rep. 528b*), and according to Burnet we have express testimony that the five Platonic figures were discovered in the Academy.³ In the *Scholia to Euclid* we read that the Pythagoreans only knew the cube, the pyramid (tetrahedron) and the dodecahedron, while the octahedron and the icosahedron were discovered by Theaetetus,⁴ the creator of solid geometry who was possibly a member of the Academy. In fact Celtic and Etruscan dodecahedra of considerable antiquity are found in the Louvre and elsewhere, which Burnet finds significant in view of the suggested connections between Pythagoreanism and the North. Critchlow is of the opinion that it is likely that the *Timaeus* was written to enable the reader to "sit in" on a more ancient oral tradition,⁵ but that Plato was obviously guarded as to how much he revealed of the mathematical knowledge of the Pythagorean tradition.⁶

All agreed that it was Pythagoras who systematized geometry and transported it from Egypt to Greece.⁷ Isidore of Seville maintained that the discipline of geometry was first discovered by the Egyptians, who derived it in order to measure land after the periodic inundation by the waters of the Nile.⁸ Like each of the disciplines that were to be developed in the Quadrivium, there were two kinds of

geometry - speculative and practical. Regular solids were known of in the Old Kingdom: icosahedral dice have been discovered. The geometric proportion used in delineating Egyptian architecture and ornament was highly sophisticated with particular import being placed on Golden Proportion $\phi = (\sqrt{5} + 1) / 2 = 1.618\dots$ They were surely familiar with all of the regular solids.

Aristotle offered a snide critique of Plato's work in the *De caelo*, and later in that work concerns himself with the Pythagorean belief "that the centre (of the universe) is occupied by fire, and that the earth is one of the stars, and creates night and day as it travels in a circle about the centre." (293a21-293a23).

Archimedes' work on the semi-regular solids has not survived, but as a measure of respect they are attributed to him as the Archimedean solids. Heron states Archimedes ascribed the cuboctahedron to Plato.⁹ Euclid gave geometry an authoritative codification, and may have incorporated a finished treatise by Theaetetus on the regular solids into his *Elements*. In the final book, he describes all of the semi-regular solids in a purely geometrical way. He shows how to construct them, proves they can each be inscribed in a sphere, finds all the circumradii of their containing spheres, and demonstrates the duality of the octahedron and the cube, and of the icosahedron and the dodecahedron.¹⁰ Hypsicles continued the geometry of the regular solids and demonstrated how to inscribe them one inside another. His treatise was regularly printed as Books XIV and XV of Euclid.¹¹

Plutarch, Diogenes Laertius and Stobaeus recorded the Platonic correspondence between the elements and the "mundane figures" given in the *Timaeus*. The Pythagorean doctrine as it had evolved academically in classical times was codified by Boethius - whose last noble work was written in prison while under the sentence of death - and transmitted to the Middle Ages as the Quadrivium.

During the Italian Renaissance, the painter Piero della Francesca renewed interest in the regular solids, his treatise being translated by Luca Paccioli and printed as the third and final section of his *Divina proportione*.¹² Flussas wrote an important treatise on the Platonic solids, and Thomas Digges adjoined to his father's work a further treatise.¹³

Henry Billingsley in his commentary on Euclid indicates the strong influence the regular solids had on cosmological speculation by their identification with the four elements. They were the archetypal numbers.

“These five solides now last defined, namely a Cube, a Tetrahedron, an Octohedron, a Dodecahedron and an Icosahedron are called regular bodies. As in plaine superficiees, those are called regular figures, whose sides and angles are equal, as are equilater triangles, equilater pentagons, hexagons, & such lyke, so in solides such only are counted and called regular, which are comprehended under equal playne superficiees, which have equal sides and equal angles, as all these five foresayed have, as manifestly appeareth by their definitions, which were all given by this proprietie of equalitie of their superficiees, which have also their sides and angles equal. And in all the course of nature there are no other bodies of this condition and perfection, but onely these five. Wherefore they have ever of the auncient Philosophers bene had in great estimation and admiration, and have bene thought worthy of much contemplacion, about which they have bestowed most diligent study and endeavour to searche out the natures & properties of them. They are as it were the ende and perfection of all Geometry, for whose sake is written whatsoever is written in Geometry. They were (as men say) first invented by the most witty Pithagoras then afterward set forth by the divine Plato, and last of all mervelously taught and declared by the most excellent Philosopher Euclide in these bookes following, and ever since wonderfully embraced of all learned Philosophers. The knowledge of them containeth infinite secretes of nature. Pithagoras, Timaeus and Plato, by them searched out the composition of the world, with the harmony and preservation therof, and applied these five solides to the simple partes therof, the Pyramis, or Tetrahedron they ascribed to the fire, for that it ascendeth upward according to the figure of the Pyramis. To the ayre they ascribed the Octohedron, for that through the subtle moisture which it hath, it extendeth it selfe every way to the one side, and to the other, accordyng as that figure doth. Unto the water they assigned the Icosahedron, for that it is continually flowing and moving, and as it were makynge angles on every side according to that figure. And to the earth they attributed a Cube, as to a thing stable, firme and sure as the figure signifieth. Last of all a Dodecahedron, for that it is made of Pentagons, whose angles are more ample and large then the angles of the other bodies, and by that meanes draw more to roundnes, & to the forme and nature of a sphere, they assigned to a sphere, namely, to heaven. Who so will read Plato in his Timaeus, shall read of these figures, and of their mutuall proportion, straunge matters, which here are not to be entreated of, this which is sayd, shall be sufficient for the knowledge of them, and for the declaration of their diffinitions.”¹⁴

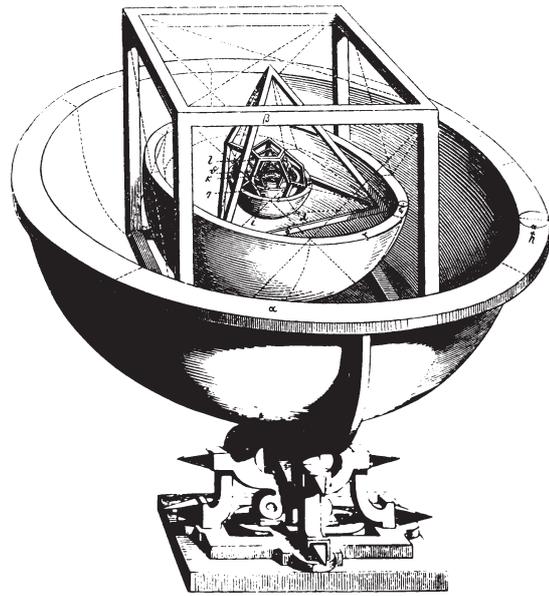


Figure 1: Kepler's mysterium cosmographicum

Such speculation culminated in the cosmological theories of Kepler,¹⁵ who argued in his *Mysterium cosmographicum* that the intervals between planets were determined by the distances between spheres circumscribing the regular solids as they are placed concentrically. Near the end of his career, he was still obsessed with the notion that the regular solids were the archetypal forms in the mind of the creator, and in the *Harmonices mundi libri V* he expounded their characteristics and virtues.

SYMBOLISM AND STRUCTURE OF SOME HISTORICAL ORDERS OF THE POLYHEDRA

We see that attempts have been made throughout history to relate the polyhedra into a comprehensive order infused with cosmic symbolism. These have almost exclusively concentrated on the regular solids.

Two kinds of relationship of the regular solids are readily identifiable: the Pythagorean cosmology given by Plato in the *Timaeus*, and a model attributed to Hindu mythology which has recently been described by Robert Lawlor. In addition, Keith Critchlow has developed the Pythagorean cosmology in accord with the Holy Tetractys. I propose to critique these orders in terms of their geometric and formal appropriateness, before discussing my work.

The Pythagorean or Platonic model has been well described by Critchlow,¹⁶ where the four elements are ascribed to four of the Platonic polyhedra, whilst the fifth is alluded to as their quintessence and represents the encompassing firmament. The quote by Henry Billingsley (above) indicates that the reasoning beyond the assignments of polyhedra to elements was not arbitrary; the properties of the polyhedra do correlate empirically, though selectively, with the complexions of the elements.

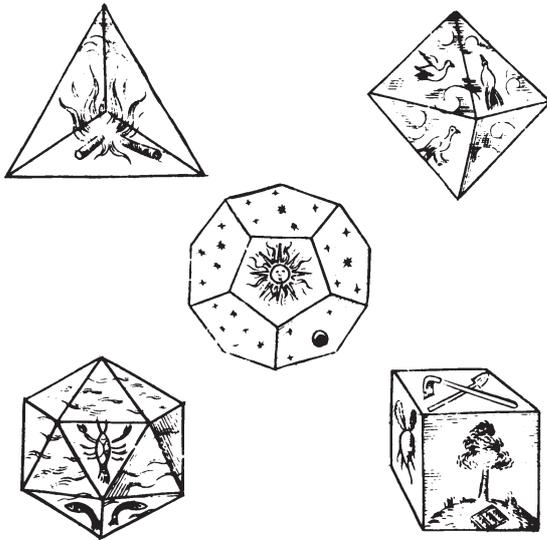


Figure 2 : The five elements as the aedicule or the centered four, after Kepler, Harmonices mundi libri V

The basis pattern of the solids is therefore the tetrad or quaternion, developing into the five as the centered four or the aedicule:

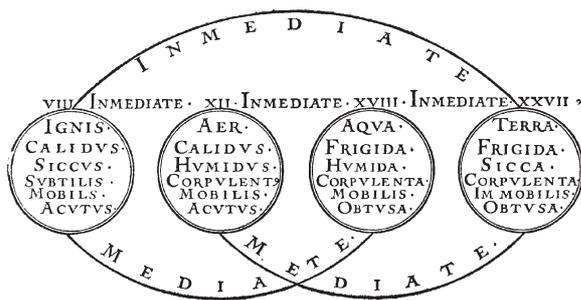


Figure 3 : The four elements arranged in arithmetical progression, Bede, "De natura rerum" in Opera (Basle, 1563), II.5

The primary axis is developed from the opposition between the tetrahedron of fire and the cube of earth. But these extremes need to be put into proportional relationship with one another through intermediary elements of the octahedron of air and the icosahedron of water. The basic linear sequence is given by the relative qualities of the cosmological elements; accordingly the 'maker' "set water and air between fire and earth, and made them so far as was possible, proportional to one another, so that as fire is to air, so is air to water, and as air is to water, so is water to earth, and thus he bound together the frame of a world visible and tangible." (Timaeus 32b).

Whilst credence is given to the particular ascription of elements to solids, their geometric interrelationship is subordinate to the notion of a linear proportional sequence. This may be understood as a cyclic progression as fire condenses to air, air to water, and water to earth; but conversely, earth rarefies to water, water to air, and air to

fire, so that despite the continuous mutation the net change is zero. Nature remains a constant, endlessly repeating this circular pattern.¹⁷ Although there is partial recognition of this continuous movement in the sequence of like triangles developing tetrahedron, octahedron and icosahedron, the pattern sits somewhat uneasily with the inherently non-linear geometrical structure.

The elements are also distributed as a quaternion of paired dualities in two ways. Critchlow preserves the primary vertical axis of fire-earth as expansion-contraction, and develops the horizontal axis as water-air.¹⁸ But in neither opposition, of tetrahedron-cube and icosahedron-octahedron, do these accord with the natural geometric dualities of tetrahedron-tetrahedron, octahedron-cube, and icosahedron-dodecahedron.

The alternative quaternion is obtained by sharing, to give air-earth as the vertical axis and fire-water as the horizontal, as in the case of the squared circle of Rosicrucian medicine, or traditional fourfold Islamic symbolism.¹⁹ Whilst the vertical air-earth polarity of octahedron-cube is dual, the horizontal polarity of fire-water, i.e. tetrahedron-icosahedron is not. But adjacent elements share a quality: fire-air are hot; air-water are moist; water-earth are cold; and earth-fire are dry. Moreover, earth reconciles the opposites of water and fire. The cyclic progression fire-air-water-earth is somewhat more satisfactory, in at least preserving the natural tetrahedron-octahedron-icosahedron progression.

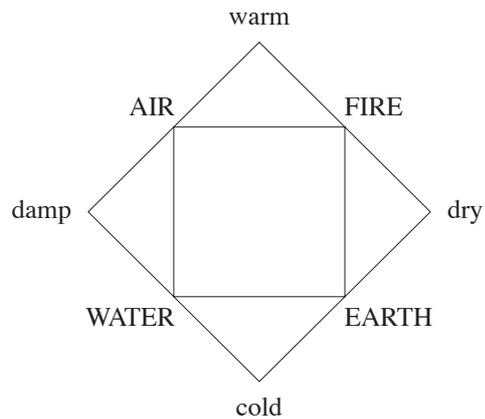


Figure 4 : The quaternion of elements with shared qualities

The elements are then arranged concentrically in accord with both their natural geometric interrelationship and a geocentric cosmological structure, with the dodecahedron being ascribed to the heavens. The construction and symbolism of this concentric model is fully described by Critchlow,²⁰ who regards it as a macrocosmic model based on the idea of an element compressing down into its own sphere, until the whole is completely filled - the whole representing the cosmos.

The only other reasonably logical order that could be advanced would appear to be the inverse sequence, but this does not make a great deal of geometric sense, being difficult to construct. As far as I am aware, the semi-regulars are not integrated into this order either.

In a modern work,²⁵ Critchlow advances an order for the regular and semi-regular polyhedra that has mythological significance. He first arranges the regular bodies as five faceted solids and five spherical polyhedra according to the Holy Tetractys.²⁶ The Tetractys has considerable strength as an organizing schema in that it relates many diverse aspects of number, space, time and meaning. Proclus says:

*- Sacred Number springs
From th'uncorrupted Monad, and proceeds
To the Divine Tetractys, she who breeds
All; and assigns the proper bounds to all,
Whom we the pure immortal Decad call.²⁷*

The Tetractys was an essential part of the Quadrivium which, according to cabalistic tradition, was preserved by inscriptions on the pillars of Seth when God destroyed the world by flood and flame. But the application of the Tetractys here does less justice to the natural energetic relationships of the regular polyhedra. He then arranges the semi-regulars in cuboctahedral array about a central truncated tetrahedron, then removes that central body, and disposes the duals of the remaining twelve semi-regular polyhedra in icosahedral array, with two families of symmetry separated out with the duals of the six truncations of the octahedron above and the six truncations of the icosahedron below.²⁸ Although of considerable interest and value, these patterns do not do full justice to the natural geometrical interrelationships of the semi-regular solids. However, these efforts of Critchlow are certainly in the spirit of the structurings of these polyhedra which we would expect to have been developed by the Pythagoreans and within Plato's Academy. If only we had access to Archimedes' treatise!

Later in the work, Critchlow schematizes the polyhedra in a periodic arrangement of elements of spatial order.²⁹ He develops a twofold sequence of truncations of the parent regular pairs octahedron-cube and icosahedron-dodecahedron, in which corresponding truncated pairs rigorously correspond. But the linear sequences of semi-regulars as progressive truncations are unsatisfactory; and the tetrahedron and truncated tetrahedron remain as "nuclear" or "over" solids.

Again, there is a sense in which the polyhedral elements are forced into an overall pattern that does not fully respect their inherent harmony of interrelationship.

MYTHIC INTERPRETATION

So we come to what I present as a new order in space, which is to be published in full detail in the International Journal of Space Structures.³⁰ I believe this does fully respect the richness of interrelationship one to another that the polyhedra demonstrate to the contemplative eye; they represent as it were gifts of the intelligence of the heart.

Here the cosmological mythology tends further to take its measure from the divine patterns revealed. This is not to subordinate the mythical - or the spiritual or divine - to its manifestation in the world of phenomena. It is rather to recognize that the natural harmony that is revealed when events are situated in their correct perspective, is symbolic of that higher harmony, beauty and order that regulates the affairs of man and cosmos.

I contemplated the perfect forms of the Eternal, responded to them, but was troubled by the inadequacy or imperfection of pattern relating them. At that time the five could best be understood as a lambda centered on the tetrahedron, thus:

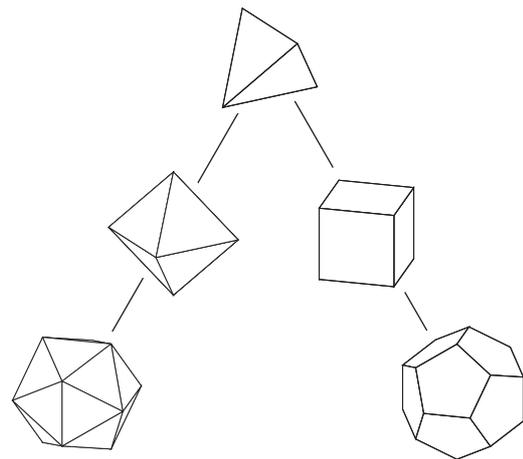


Figure 7 : Platonic lambda of polyhedra

But to contemplate the lack of pattern of the semi-regular forms was to confront chaos. Despite isolated instances of relationship between two or three solids at a time, there seemed no satisfying overall order relating them all one to another. The closest approximation to such a pattern was Keith Critchlow's order, with paired linear truncation sequences of the octahedron-cube and icosahedron-dodecahedron, and with the tetrahedron and truncated tetrahedron as over solids. But close scrutiny of this indicated the unsatisfactoriness of the strict monolinear order of either truncation sequence.

A process of discovering natural order followed. In playing with possible patterns, limited truncation sequences were evident progressing between regular dual poles. The quasi-regulars seemed important as being midway between these extremes of the regulars; and a natural correlation seemed apparent between the quasi-regulars,

small rhombs, and great rhombs of what were to be accorded to Classes II and III of the new order. The snubs, appropriately, were off by themselves, appearing at that time to be a special case.

It eventually made sense to structure the polyhedra along two axes, by polarizing the fundamental perfect dualities along the horizontal axis, and developing two orthogonal sequences from the quasi-regulars along the vertical axis. This gave a sensible contracted form, which was then extended to incorporate the snubs and the truncated poles in a meaningful position.

At this time it was possible to deduce the existence of a third class, displaying the symmetries of the tetrahedron and truncated tetrahedron. The order could then be rigorously developed in its threefold nature, with the recognition of certain polyhedra being repeated with alternative symmetries. The deduction of these solids employed a similar mode of reasoning to that of the scientists of the last hundred years who filled in the periodic table of elements. Thus the octahedron as male pole of Class II reappeared as the quasi-regular neutral center of Class I; the icosahedron as male pole of Class III reappeared as the snub polyhedron of Class I; the cuboctahedron as quasi-regular neutral center of Class II reappeared as the small rhomb of Class I; the truncated octahedron as truncated male pole of Class II reappeared as the great rhomb of Class I; and within Class I both tetrahedron and truncated tetrahedron reappeared in alternative orientation as polar and truncated polar solids respectively.

The cross-class regularities revealed a perfection and harmony that was supra-mundane. Unexpected correspondences were revealed allowing interpolations and extrapolations, and these checked out perfectly every time. Finally, it was found meaningful to rigorously extend the order into two-dimensions to include all but one of the regular and semi-regular tilings of the plane. (I consider the exception degenerate, with good reason).

The new order sustains prolonged contemplation, and continues to generate further integrative discoveries. It has a "rightness" about it that continues to astound and to delight. But in so doing it raises questions and provokes insights into the mythic realm of perfection.

The regular polyhedra have been regarded through the ages as perfect forms. In a very real sense they are given - pure exemplars of the archetypes dwelling beyond the realm of mundane appearances. They represent revealed truth of the intrinsic nature of pure space. But in contemplating these perfect forms, as individual centers, one seeks to comprehend the pattern relating them one to another and to the less perfect semi-regular forms. One inevitably becomes aware of the inherent duality between the regular forms, which leads one to posit that space has an intrinsic quality of three-foldness, in that it supports just three fundamental symmetry patterns. At the same time, it exhibits through each class an inherent duality. The octahedral-cubic and icosahedral-dodecahedral dualities taken together with the self-dual nature of the

tetrahedron suggest that there are not five perfect solids, but six, being the three pairs of regular solids (one pair being identical, but standing in opposite orientation). There is a definite sequence through the three classes, from simplicity to complexity, i.e. from tetrahedral-tetrahedral, through octahedral-cubic, to icosahedral-dodecahedral.

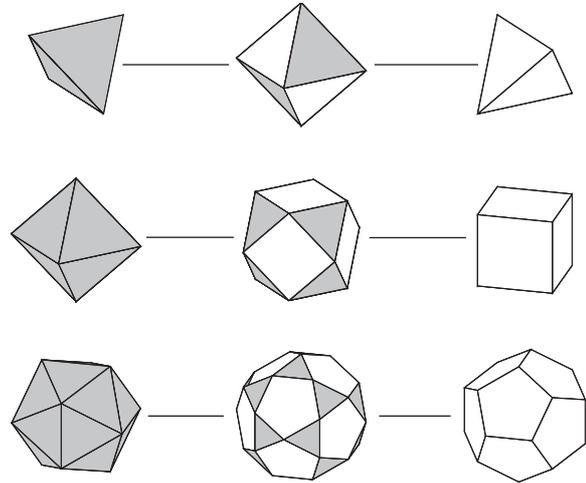


Figure 8 : The quasi-regular polyhedra as the perfectly harmonized centers of the polar duals of Platonic regular polyhedra

But in so considering the regular bodies, it becomes obvious that any one of them - though perfect in itself - does not represent the perfect center. It is not a balanced form, but rather one perfect pole of a duality. The perfect forms are therefore extremes. They are perfect forms only in the realm of duality. This suggests that the quasi-regular polyhedra - the tetratetrahedron (i.e. octahedron), cuboctahedron, and icosidodecahedron are the perfectly balanced centers. They precisely mediate and harmonize the polar extremes. They can be considered to give birth to, to give rise to all of the regular and semi-regular polyhedra of their respective class.

So in a very real sense, contemplation of perfection in relationship to its fellows indicates an inherent duality, which reveals its extreme polar nature. But contemplative penetration into the center that lies between the paired expressions of perfection reveals the more perfect source which contains within its harmonized form the potential to give rise to all the possibilities inherent in that symmetry system.

This line of reasoning can be developed. The recognition of neutral quality lying between polar extremes raises into awareness a simultaneous triadic structure to the recognized dualism. In the regular poles and their truncated solids either positive or negative quality predominates. At the level of duality, therefore, the quasi-regular evolves in polar directions along the horizontal axis through the intermediary truncate forms to reach those polar regular solids. In the remaining semi-regular polyhedra however these polar qualities are balanced, and therefore these solids are ordered on the vertical axis. The

quasi-regular center, through dynamic expansion and rotation, generates firstly the snub polyhedra as a transitional form, and completes that lesser development in the small rhombic solid which arises as a more evolved manifestation of positive, neutral and negative harmony. A greater development is the transcendence sequence which evolves from the quasi-regular, through the small rhombic solid, to culminate in the great rhombic solid. The great rhombic solid represents for each class the most fully developed expression of the potentiality that lies concealed within the initial realization of perfection.

In Neoplatonic terms, by turning away from the realm of duality, Man can ascend in spirit from the level of the body to the level of universal Soul, become whole, and in Soul attain to Intellect.

Finally, the “pattern that connects” is found in comprehension of the beautiful and complete triadic structure of the entire order, which reveals the wondrous coherent and articulate nature of the void.

- 1 Critchlow, Keith, *Time Stands Still - New Light on Megalithic Science*. Gordon Fraser, London 1979. See in particular Chapter 7: *Platonic Spheres - a millennium before Plato*, pp.131-149.
- 2 Burl, Aubrey, *The Stonehenge People*. Guild Publishing, London, 1987, p.103.
- 3 Burnet, John, *Early Greek Philosophy*. Adam & Charles Black, London, 1948, p.283.
- 4 Heiberg's *Euclid*, vol. v. p.654, I, referenced in Burnet, *ibid.*, p.284, ref.1.
- 5 Critchlow, K., *Time Stands Still*, op. cit., p.134.
- 6 Critchlow, Keith, *Islamic Patterns - An Analytical and Cosmological Approach*. Thames and Hudson, London, 1976, p.105.
- 7 In my historical review I follow closely: Heninger, S.K. Jr., *Touches of Sweet Harmony - Pythagorean Cosmology and Renaissance Poetics*. The Huntington Library, San Marino California, 1974.
- 8 *Ibid*, p.106.
- 9 Critchlow, Keith, *Order in Space - a Design Source Book*. Thames and Hudson, London, 1969, p.32, n.1.
- 10 *Ibid*, pp.20, 34.
- 11 Heninger, op. cit., p.109.
- 12 *Ibid*, p.109.
- 13 *Ibid*, p.109.
- 14 *Elements*, fol. 319^v-320, quoted in Heninger, *ibid*, pp.109-10.
- 15 *Ibid*, pp.110-11.
- 16 Critchlow, Keith, *The Platonic Tradition on the Nature of Proportion*, in *Lindisfarne Letter #10 - Geometry and Architecture*. The Lindisfarne Association, Lindisfarne Press, W. Stockbridge, 1980.
- 17 Heninger, op. cit., p.49.
- 18 Critchlow, K., *The Platonic Tradition on the Nature of Proportion*, op. cit., p.17.
- 19 See for example Burckhardt, Titus, *Moorish Culture in Spain*. McGraw-Hill Book Company, London 1972, p.68.
- 20 Critchlow, K., *The Platonic Tradition on the Nature of Proportion*, op. cit., pp. 29-32.
- 21 *Ibid*, p.19.
- 22 For a clear Islamic perspective on this see Nasr, Seyyed Hossein, *Man and Nature - The Spiritual Crisis of Modern Man*. A Mandala Book, Unwin, London, 1976.
- 23 Lawlor, Robert, *Sacred Geometry - Philosophy and Practice*. Thames and Hudson, London, 1982, pp. 98-103.
- 24 *Plummer, Gordon, The Mathematics of the Cosmic Mind*, referred to in Lawlor, *ibid.*, p.96.
- 25 Critchlow, K., *Order in Space*, op. cit.
- 26 *Ibid*, pp. 22-3.
- 27 Translated by Thomas Stanley, and quoted in Heninger, op. cit., p.84.
- 28 *Ibid*, pp. 38-9.
- 29 *Ibid*, Appendix One: *A Periodic Arrangement of the Elements of Spatial Order*.
- 30 Meurant, Robert C., *A New Order in Space - Platonic and Archimedean Polyhedra and Tilings*. *International Journal of Space Structures*, Vol.6 No.2, University of Surrey, 1991.