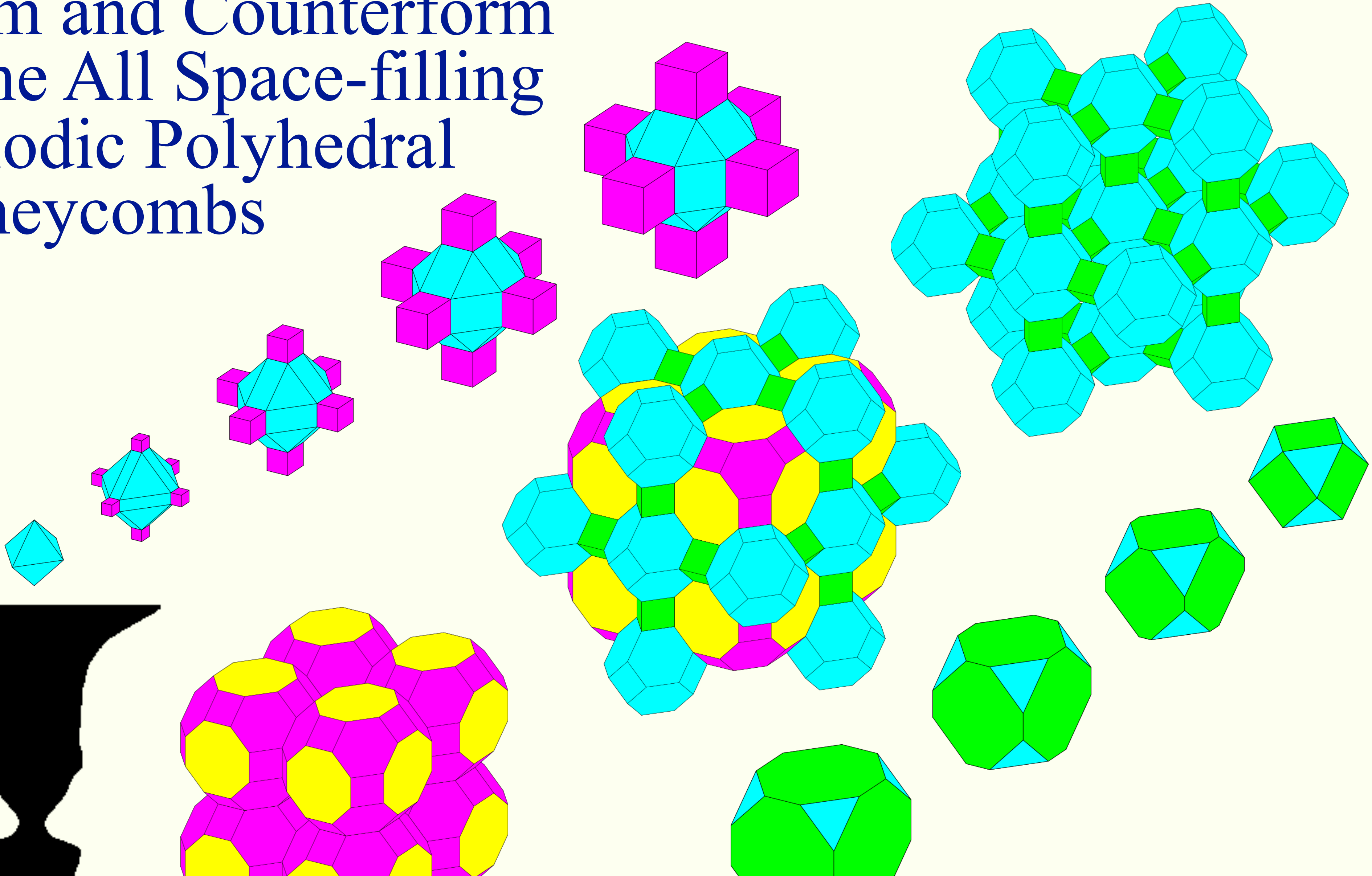
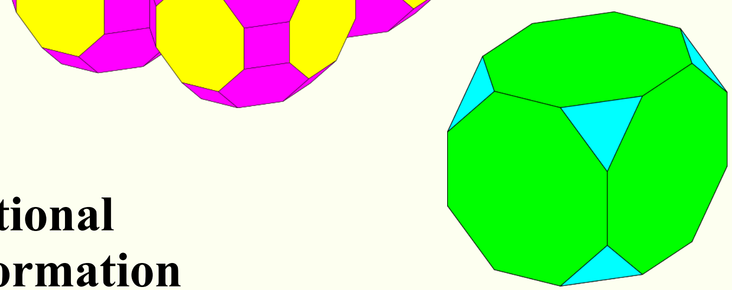
Form and Counterform in the All Space-filling Periodic Polyhedral Honeycombs





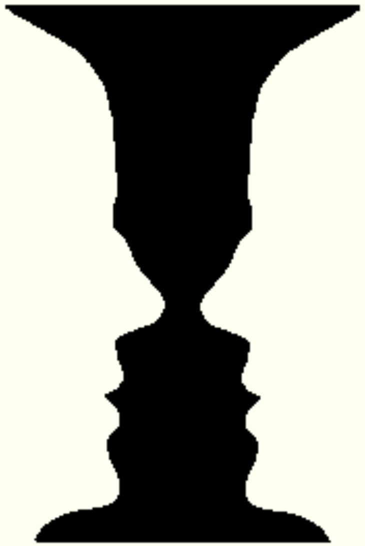


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Table 1. Primary Polyhedral Vertices; Edges, Faces & Areas by Axes & Overall; and Volumes.

Polyhedron	Vertices $\sum$	Edges $\sqrt{1}$	Edges $\sqrt{2}$	Edges $\sum$	3-gon $\sqrt{3}$	3-gon $\sqrt{3}$ rot	4-gon $\sqrt{1}$	4-gon $\sqrt{1}$ rot	4-gon $\sqrt{2}$	6-gon $\sqrt{3}$	8-gon $\sqrt{1}$	Faces $\sum$	Area $\sqrt{1}$	Area $\sqrt{2}$	Area $\sqrt{3}$	Area $\sum$	Volume
T	4	-	6	6	-	4	-	-	-	-	-	8	-	-	$4\sqrt{3}$	$4\sqrt{2}$	$2\sqrt{2}/3$
D	12	-	18	18	4	-	-	-	-	4	-	8	-	-	$4\sqrt{3}(1+6)$	$28\sqrt{3}$	$46\sqrt{2}/3$
OH	6	-	12	12	8	-	-	-	-	-	-	8	-	-	$8\sqrt{3}$	$8\sqrt{3}$	$8\sqrt{2}/3$
TO	24	-	36	36	-	-	-	4	-	8	-	12	24	-	$48\sqrt{3}$	$24(1+2\sqrt{3})$	$64\sqrt{2}$
CO	12	-	24	24	-	8	-	6	-	-	-	14	24	-	$8\sqrt{3}$	$8\sqrt{3}(1+\sqrt{3})$	$40\sqrt{2}/3$
TC	24	12	24	36	-	8	-	-	-	-	6	14	$48(1+\sqrt{2})$	-	$8\sqrt{3}$	$8(6+6\sqrt{2}+\sqrt{3})$	$56(3+2\sqrt{2})/3$
CB	8	12	-	12	-	-	6	-	-	-	-	6	24	-	-	24	8
SR	24	24	24	48	8	-	6	-	12	-	-	26	24	48	$8\sqrt{3}$	$8(9+\sqrt{3})$	$16(6+5\sqrt{2})/3$
GR	48	24	48	72	-	-	-	-	12	8	6	26	$48(1+\sqrt{2})$	48	$48\sqrt{3}$	$48(2+\sqrt{2}+\sqrt{3})$	$16(11+7\sqrt{2})$
SP	8	12	-	12	-	-	6	-	-	-	-	6	24	-	-	24	8
RP	8	8	4	12	-	-	-	2	4	-	-	6	8	16	-	24	8
OP	16	16	8	24	-	-	4	-	4	-	2	10	$16(2+\sqrt{2})$	16	-	$16(3+\sqrt{2})$	$16(1+\sqrt{2})$

$[GR|GR]^{OP}_{OP}$
$2\sqrt{2}+2$

$+1 \nearrow$ $\nwarrow +1$

$[TO|GR]^{OG}_{RP}$ $[GR|TO]^{RP}_{OG}$
$2\sqrt{2}+1$ $2\sqrt{2}+1$

$+1 \nwarrow$ $\nearrow +1$

$[SR|TC]^{OP}_{SP}$ $[TO|TO]^{RS}_{RS}$ $[TC|SR]^{SP}_{OP}$
$2+\sqrt{2}$ $2\sqrt{2}$ $2+\sqrt{2}$

$+1 \nearrow$ $\nwarrow +1$ $+1 \nearrow$ $\nwarrow +1$

$[OH|TC]^{OG}_{AE}$ $[SR|CO]^{RP}_{SQ}$ $[CO|SR]^{SQ}_{RP}$ $[TC|OH]^{AE}_{OG}$
$\sqrt{2}+1$ $\sqrt{2}+1$ $\sqrt{2}+1$ $\sqrt{2}+1$

$+1 \nwarrow$ $\nearrow +1$ $+1 \nwarrow$ $\nearrow +1$

$[OH|CO]^{RS}_{VT}$ $[CB|CB]^{SP}_{SP}$ $[CO|OH]^{VT}_{RS}$
$\sqrt{2}$ 2 $\sqrt{2}$

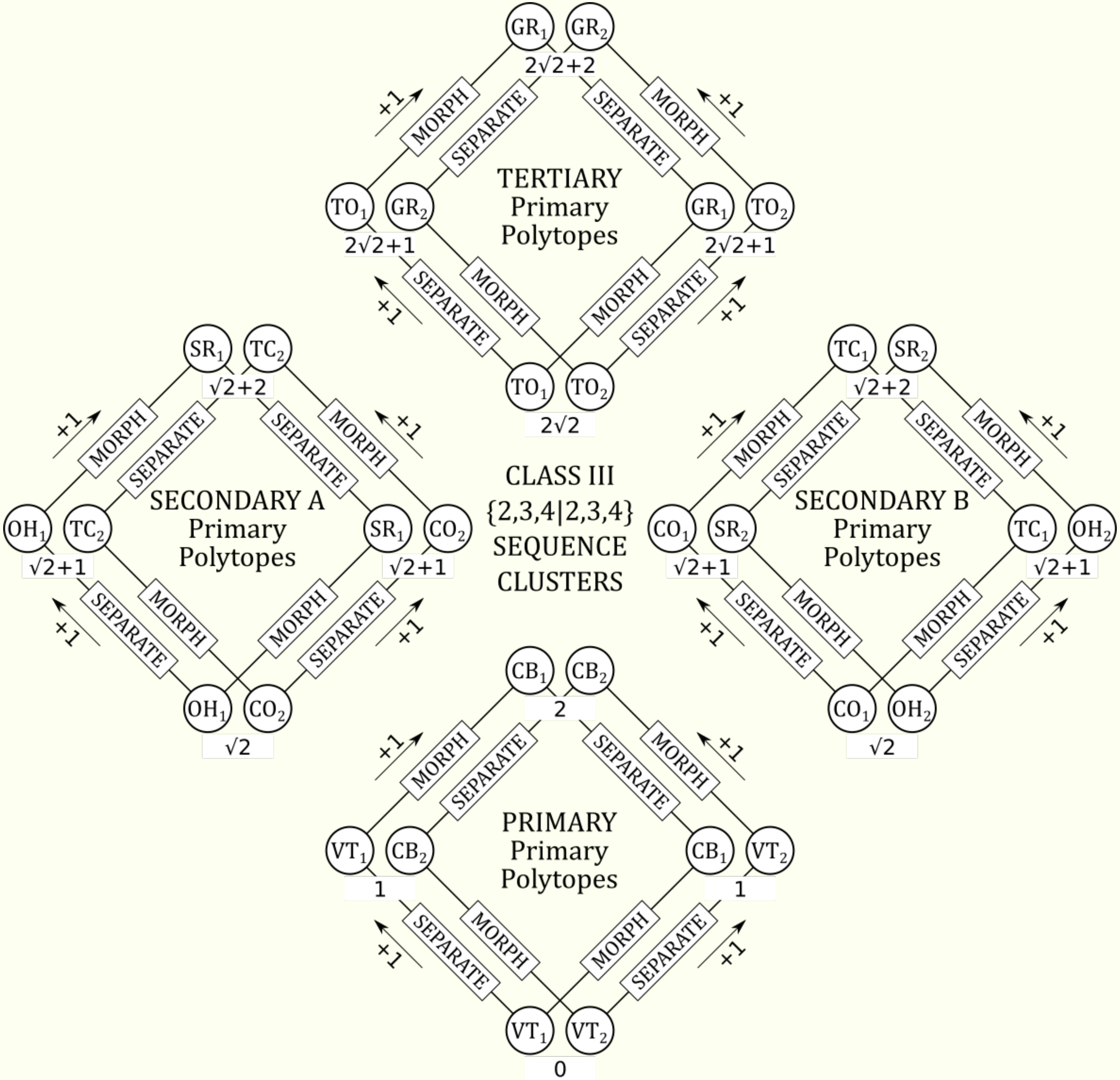
$+1 \nearrow$ $\nwarrow +1$

$[VT|CB]^{SQ}_{AE}$ $[CB|VT]^{AE}_{SQ}$
1 1

$+1 \nwarrow$ $\nearrow +1$

$[VT|VT]^{VT}_{VT}$
0

Fig. 1. The {2,3,4|2,3,4} Honeycombs as four groups of 4 contracted, transitional or expanded honeycombs, with lattice dimensions under, and unit expansion distances shown as diagonals.



CLASS III
{2,3,4|2,3,4}
SEQUENCE
CLUSTERS
TERTIARY
Primary
Polytopes
GR_1 GR_2
$2\sqrt{2}+2$
TO_1 GR_2 GR_1 TO_2
$2\sqrt{2}+1$
$2\sqrt{2}+1$
TO_1 TO_2
$2\sqrt{2}$
SECONDARY A
Primary
Polytopes
SR_1 TC_2
$\sqrt{2}+2$
OH_1 TC_2 SR_1 CO_2
$\sqrt{2}+1$
$\sqrt{2}+1$
OH_1 CO_2
$\sqrt{2}$
SECONDARY B
Primary
Polytopes
TC_1 SR_2
$\sqrt{2}+2$
CO_1 SR_2 TC_1 OH_2
$\sqrt{2}+1$
$\sqrt{2}+1$
CO_1 OH_2
$\sqrt{2}$
PRIMARY
Primary
Polytopes
CB_1 CB_2
2
VT_1 CB_2 CB_1 VT_2
1
1
VT_1 VT_2
0
MORPH
SEPARATE
+1

$$\begin{vmatrix} SR & T^+ \\ T^- & CB \end{vmatrix}\begin{vmatrix} T^- & SR \\ CB & T^+ \end{vmatrix} \qquad \begin{vmatrix} GR & D^+ \\ D^- & TC \end{vmatrix}\begin{vmatrix} D^- & GR \\ TC & D^+ \end{vmatrix}$$

$$\sqrt{2}/2+1 \quad \longrightarrow +\sqrt{2} \longrightarrow \quad 3\sqrt{2}/2+1$$

$$\begin{vmatrix} T^+ & CB \\ SR & T^- \end{vmatrix}\begin{vmatrix} CB & T^- \\ T^+ & SR \end{vmatrix} \qquad \begin{vmatrix} D^+ & TC \\ GR & D^- \end{vmatrix}\begin{vmatrix} TC & D^- \\ D^+ & GR \end{vmatrix}$$

$$+1 \uparrow \qquad \{2,3,3|2,3,4\} \qquad \uparrow +1$$

$$\begin{vmatrix} OH & T^+ \\ T^- & VT \end{vmatrix}\begin{vmatrix} T^- & OH \\ VT & T^+ \end{vmatrix} \qquad \begin{vmatrix} TO & D^+ \\ D^- & CO \end{vmatrix}\begin{vmatrix} D^- & TO \\ CO & D^+ \end{vmatrix} \qquad \begin{Vmatrix} T^+ & D^+ \\ T^- & D^- \end{Vmatrix}$$

$$\sqrt{2}/2 \quad \longrightarrow +\sqrt{2} \longrightarrow \quad 3\sqrt{2}/2$$

$$\begin{vmatrix} T^+ & VT \\ OH & T^- \end{vmatrix}\begin{vmatrix} VT & T^- \\ T^+ & OH \end{vmatrix} \qquad \begin{vmatrix} D^+ & CO \\ TO & D^- \end{vmatrix}\begin{vmatrix} CO & D^- \\ D^+ & TO \end{vmatrix} \qquad \begin{Vmatrix} D^+ & D^- \\ T^+ & T^- \end{Vmatrix} \quad \begin{Vmatrix} T^- & T^+ \\ D^- & D^+ \end{Vmatrix}$$

Fig. 3. (a) The Honeycombs of {2,3,3|2,3,4} symmetry…

$$\begin{Vmatrix} D^- & T^- \\ D^+ & T^+ \end{Vmatrix}$$

$$\sqrt{2}$$

$$\{2,3,3|2,3,3\}$$

(b) … and of {2,3,3|2,3,3} symmetry.

Table 2. Class {2,3,4|2,3,4} Lattice Sizes and Volumes by Component Polyhedra and Group

Lattice	L. size	P1 vol.	P2 vol.	3xN1 vol.	3xN2 vol.	∑ vol.
$[GR\|GR]_{OP}^{OP}$	$2\sqrt{2}+2$	$2(7\sqrt{2}+11)$	$2(7\sqrt{2}+11)$	$6(\sqrt{2}+1)$	$6(\sqrt{2}+1)$	$8(5\sqrt{2}+7)$
$[TO\|GR]_{RP}^{OG}$	$2\sqrt{2}+1$	$2(7\sqrt{2}+11)$	$8\sqrt{2}$	0	3	$22\sqrt{2}+25$
$[TO\|TO]_{RS}^{RS}$	$2\sqrt{2}$	$8\sqrt{2}$	$8\sqrt{2}$	0	0	$16\sqrt{2}$
$[SR\|TC]_{SP}^{OP}$	$\sqrt{2}+2$	$\frac{7(2\sqrt{2}+3)}{3}$	$\frac{2(5\sqrt{2}+6)}{3}$	$6(\sqrt{2}+1)$	3	$2(7\sqrt{2}+10)$
$[OH\|TC]_{AE}^{OG}$	$\sqrt{2}+1$	$\frac{7(2\sqrt{2}+3)}{3}$	$\frac{\sqrt{2}}{3}$	0	0	$5\sqrt{2}+7$
$[SR\|CO]_{SQ}^{RP}$	$\sqrt{2}+1$	$\frac{5\sqrt{2}}{3}$	$\frac{2(5\sqrt{2}+6)}{3}$	3	0	$5\sqrt{2}+7$
$[OH\|CO]_{VT}^{RS}$	$\sqrt{2}$	$\frac{5\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	0	$2\sqrt{2}$
$[CB\|CB]_{SP}^{SP}$	2	1	1	3	3	8
$[VT\|CB]_{AE}^{SQ}$	1	1	0	0	0	1
$[VT\|VT]_{VT}^{VT}$	0	0	0	0	0	0

$$Lattice\ volume\ \{2,3,4 \mid 2,3,4\} = \ (P1 + P2 + 3N1 + 3N2)$$

Table 3. Classes {2,3,3|2,3,4} and {2,3,3|2,3,3} Lattice Sizes and Volumes by Polyhedra

$\begin{vmatrix} GE^+ & P2 \\ P1 & GE^- \end{vmatrix}$	Lattice size	$\frac{P1}{2}$	$\frac{P2}{2}$	$\frac{GE^-}{2}$	$\frac{GE^+}{2}$	$\sum$ vol. = lattice size³
$\begin{vmatrix} T^+ & VT \\ OH & T^- \end{vmatrix}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{6}$	0	$\frac{\sqrt{2}}{24}$	$\frac{\sqrt{2}}{24}$	$\frac{\sqrt{2}}{4}$
$\begin{vmatrix} T^+ & CB \\ SR & T^- \end{vmatrix}$	$\sqrt{2}/2+1$	$\frac{5\sqrt{2}+6}{3}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{24}$	$\frac{\sqrt{2}}{24}$	$\frac{7\sqrt{2}+10}{4}$
$\begin{vmatrix} D^+ & CO \\ TO & D^- \end{vmatrix}$	$\frac{3\sqrt{2}}{2}$	$4\sqrt{2}$	$\frac{5\sqrt{2}}{6}$	$\frac{23\sqrt{2}}{24}$	$\frac{23\sqrt{2}}{24}$	$\frac{27\sqrt{2}}{4}$
$\begin{vmatrix} D^+ & TC \\ GR & D^- \end{vmatrix}$	$\frac{3\sqrt{2}}{2}+1$	$8\sqrt{2}$	$8\sqrt{2}$	$\frac{23\sqrt{2}}{24}$	$\frac{23\sqrt{2}}{24}$	$\frac{45\sqrt{2}+58}{4}$
$\begin{Vmatrix} D^- & T^- \\ D^+ & T^+ \end{Vmatrix}$	Lattice size	$\frac{D^+}{2}$	$\frac{T^-}{2}$	$\frac{T^+}{2}$	$\frac{D^-}{2}$	$\sum$ vol. = lattice size³
$\begin{Vmatrix} D^- & T^- \\ D^+ & T^+ \end{Vmatrix}$	$\sqrt{2}$	$\frac{23\sqrt{2}}{24}$	$\frac{\sqrt{2}}{24}$	$\frac{\sqrt{2}}{24}$	$\frac{23\sqrt{2}}{24}$	$2\sqrt{2}$

$$Lattice\ vol.\{2,3,3|2,3,4\} = \frac{P1+P2+GE^-+GE^+}{2}\ ;\ \{2,3,3|2,3,3\} = \frac{D^+}{2}+\frac{T^-}{2}+\frac{T^+}{2}+\frac{D^-}{2} = D+T$$

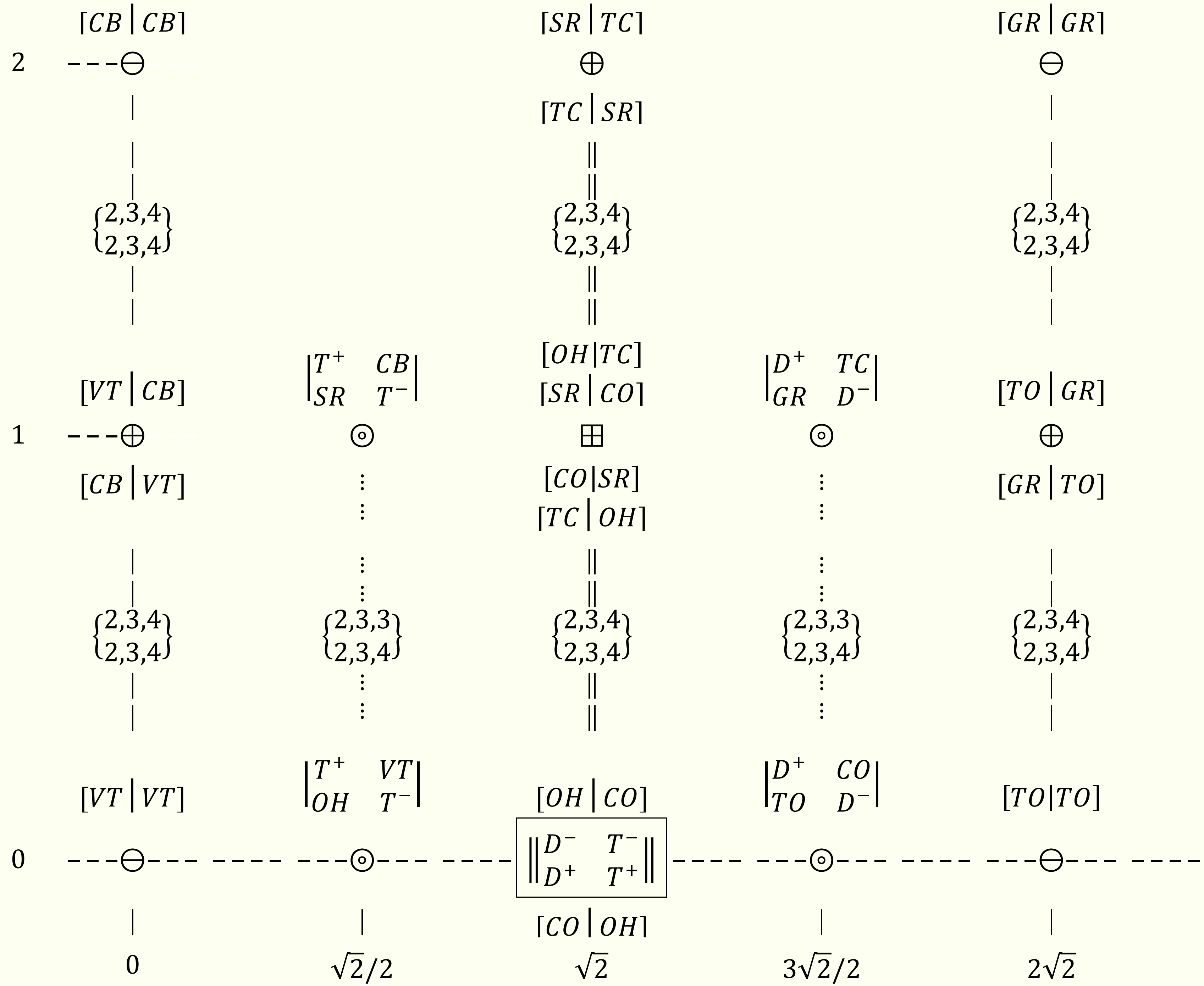


Fig. 2. Graph of Lattice sizes, showing bilateral symmetry and class-to-class correspondence.

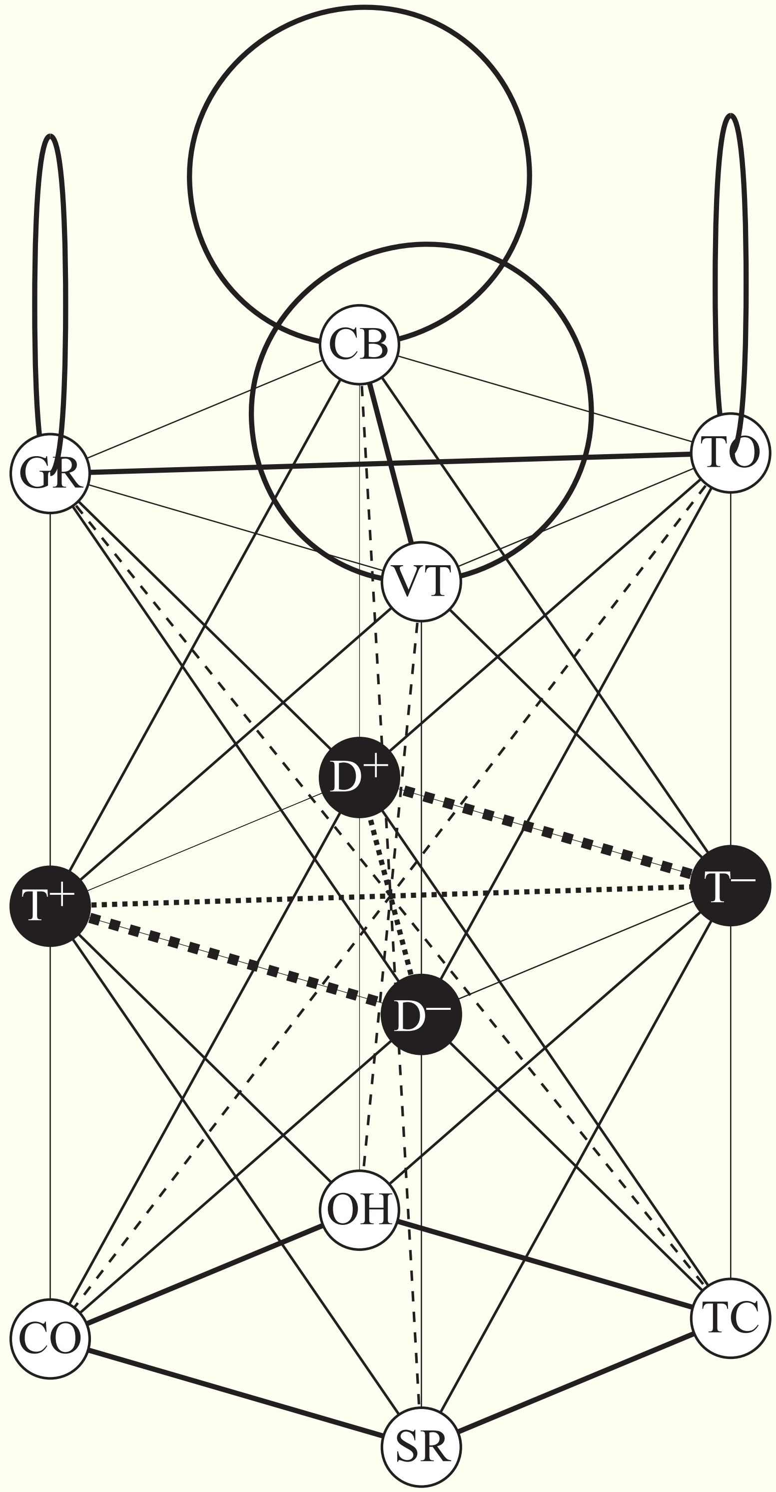
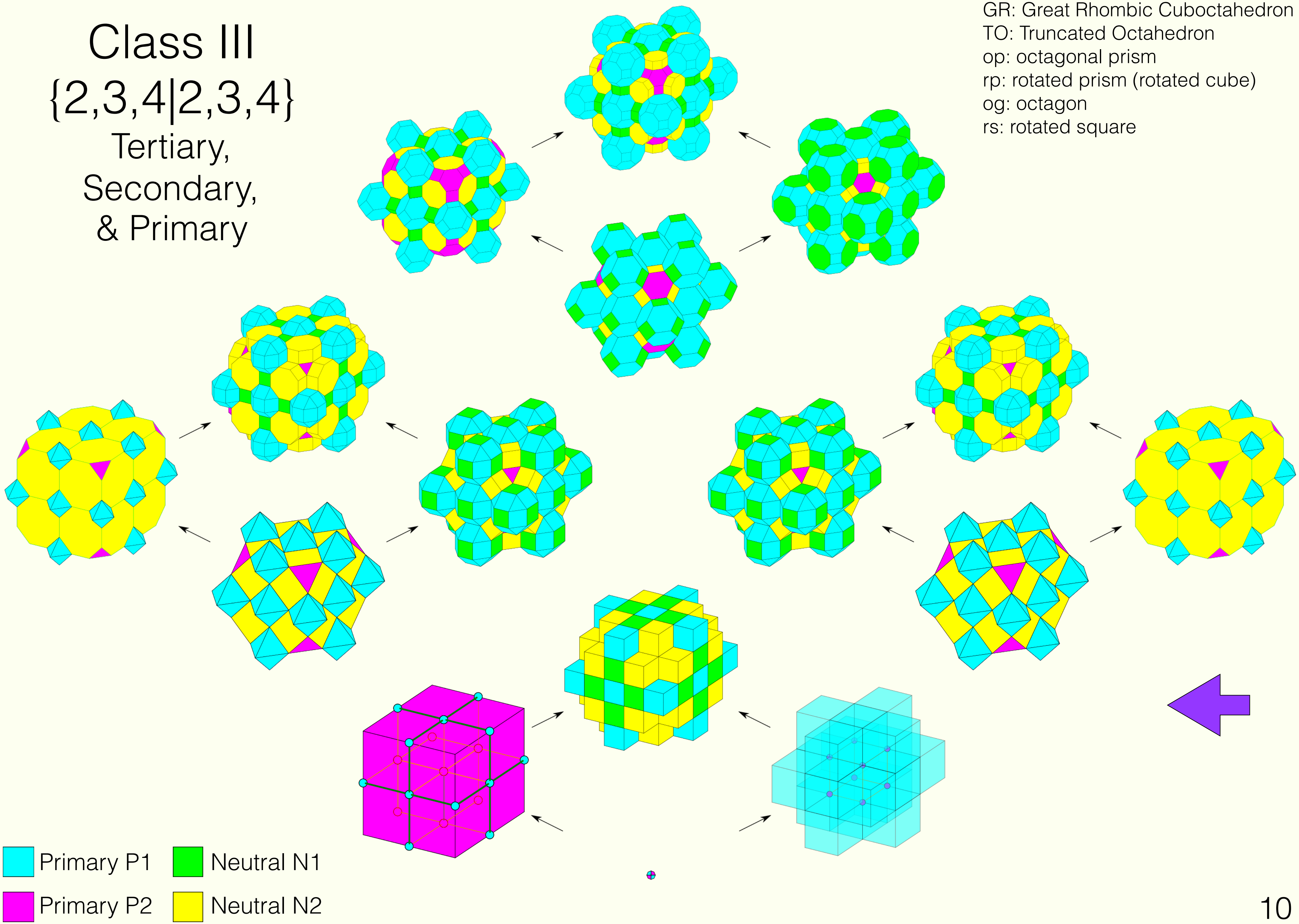


Fig. 5. Three-dimensional arrangement of the four and eight together with their linkages that reveals the meta-order of the all-space-filling periodic honeycombs.

Class III
{2,3,4|2,3,4}
Tertiary, Secondary, & Primary

GR: Great Rhombic Cuboctahedron
TO: Truncated Octahedron
op: octagonal prism
rp: rotated prism (rotated cube)
og: octagon
rs: rotated square



Class III
{2,3,4|2,3,4}
Tertiary

GR: Great Rhombic Cuboctahedron
TO: Truncated Octahedron
op: octagonal prism
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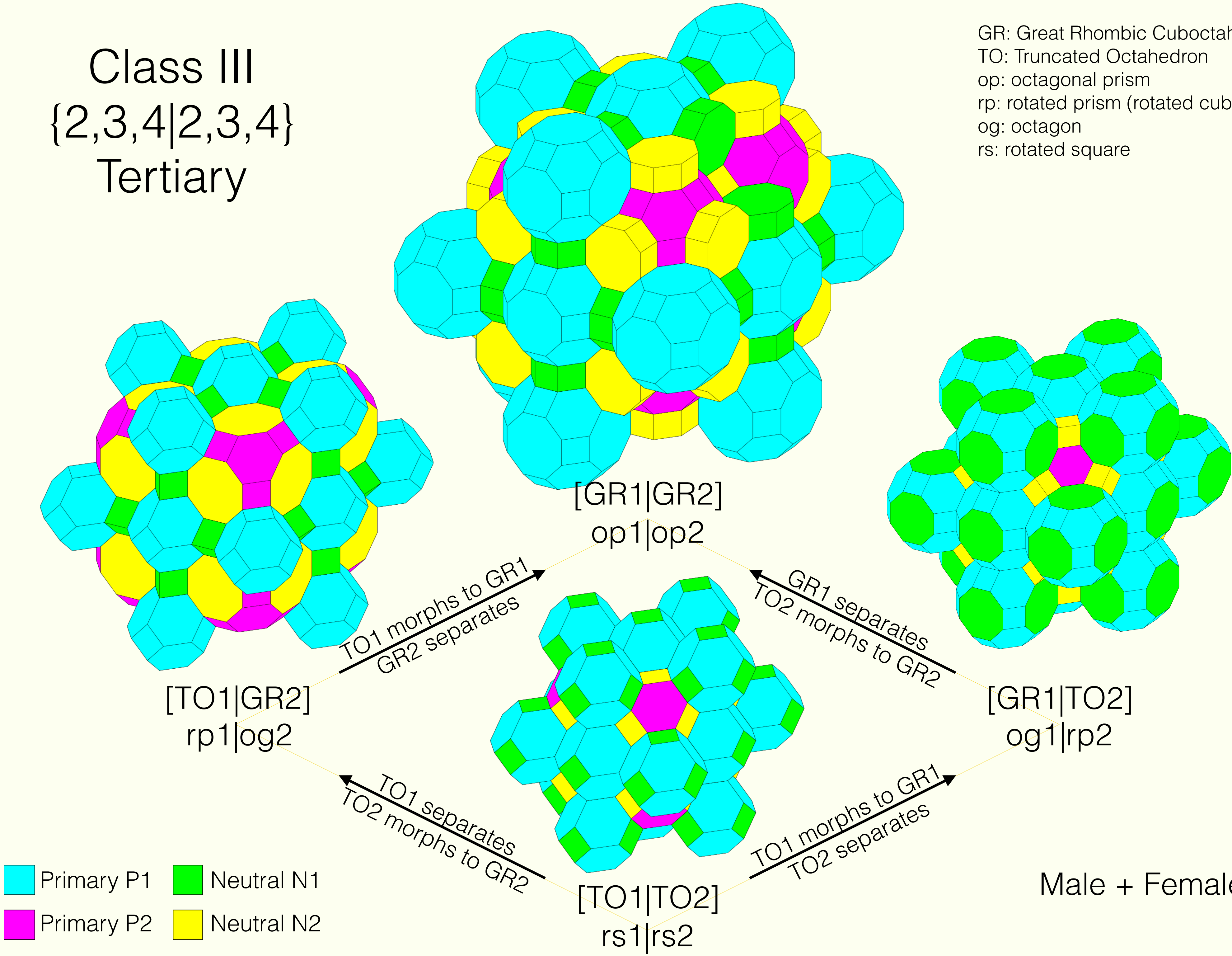
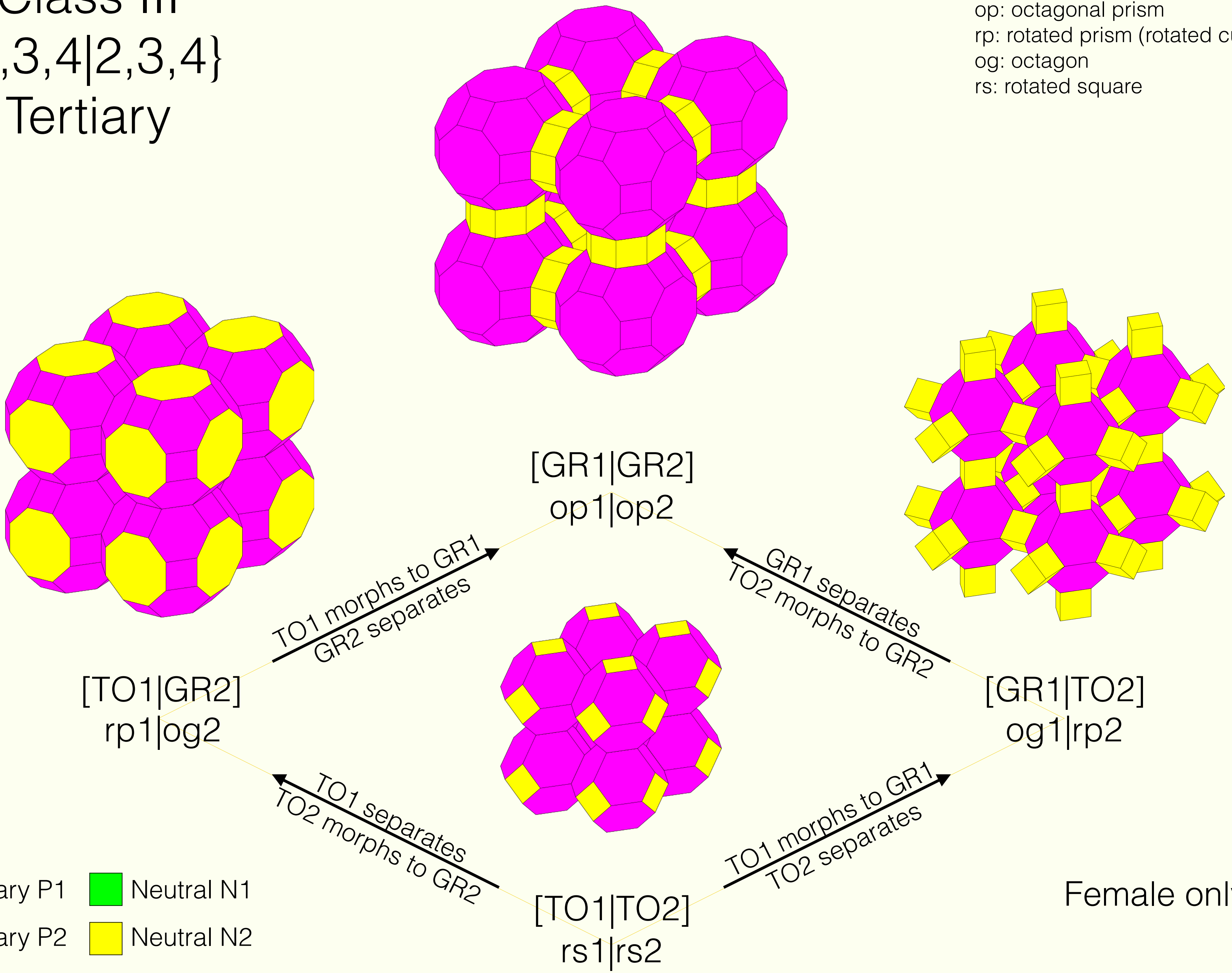
rs: rotated square



Male + Female

Class III
{2,3,4|2,3,4}
Tertiary

GR: Great Rhombic Cuboctahedron
TO: Truncated Octahedron
op: octagonal prism
rp: rotated prism (rotated cube)
og: octagon
rs: rotated square

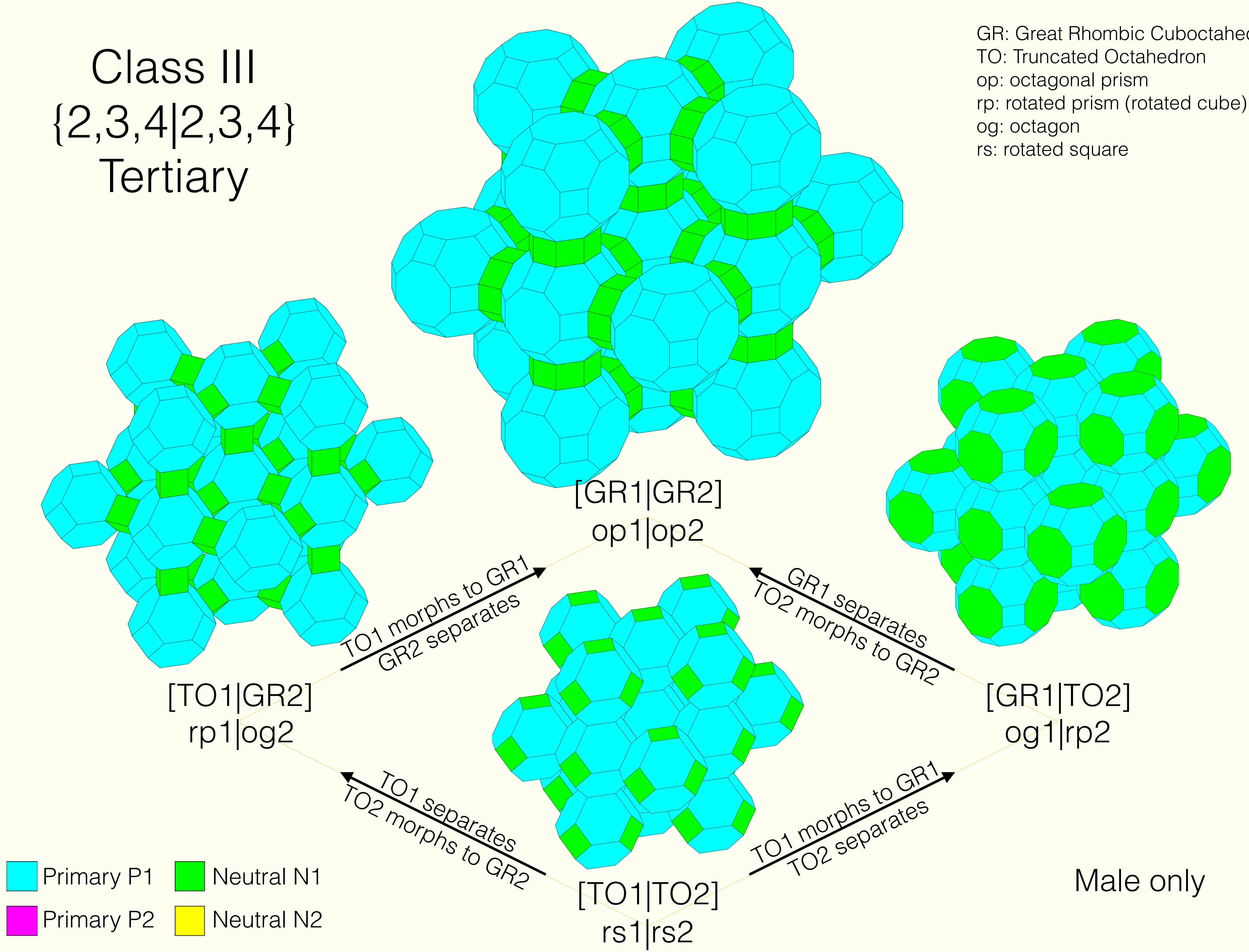


Primary P1 Neutral N1
Primary P2 Neutral N2

Female only

Class III
{2,3,4|2,3,4}
Tertiary

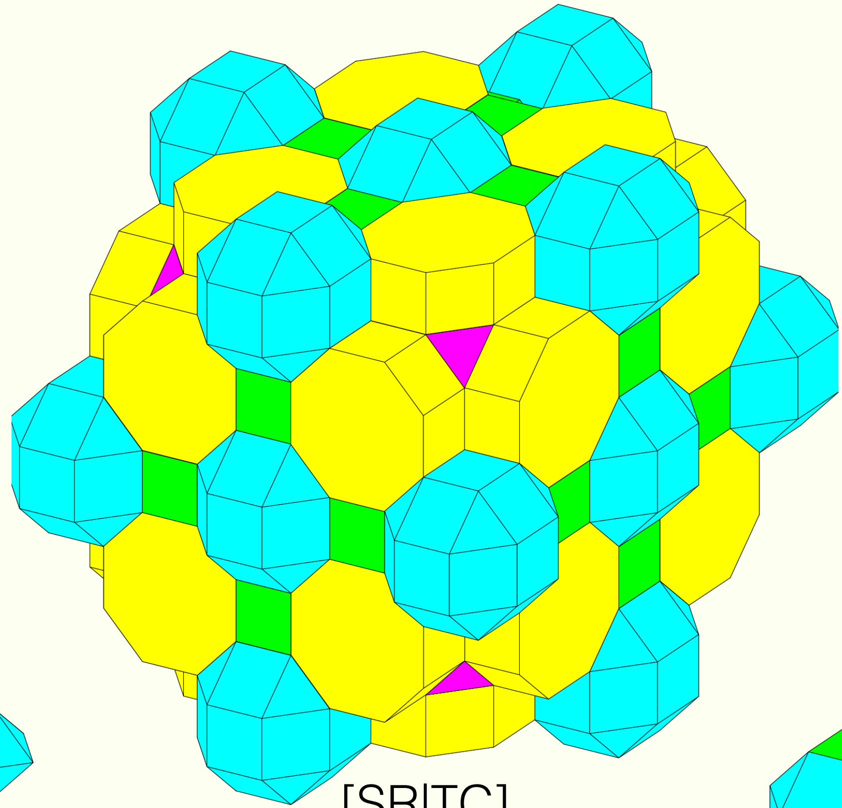
GR: Great Rhombic Cuboctahedron
TO: Truncated Octahedron
op: octagonal prism
rp: rotated prism (rotated cube)
og: octagon
rs: rotated square



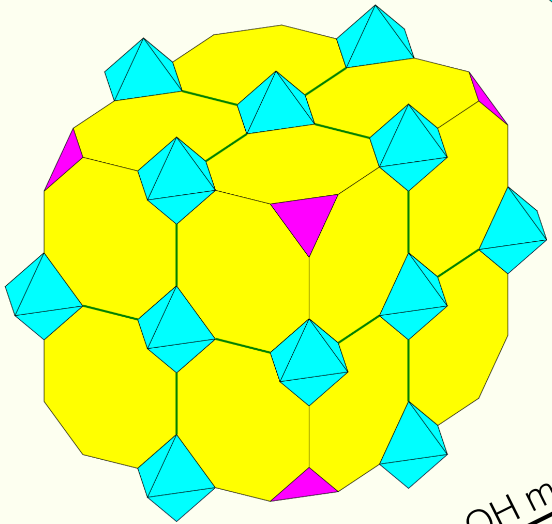
Male only

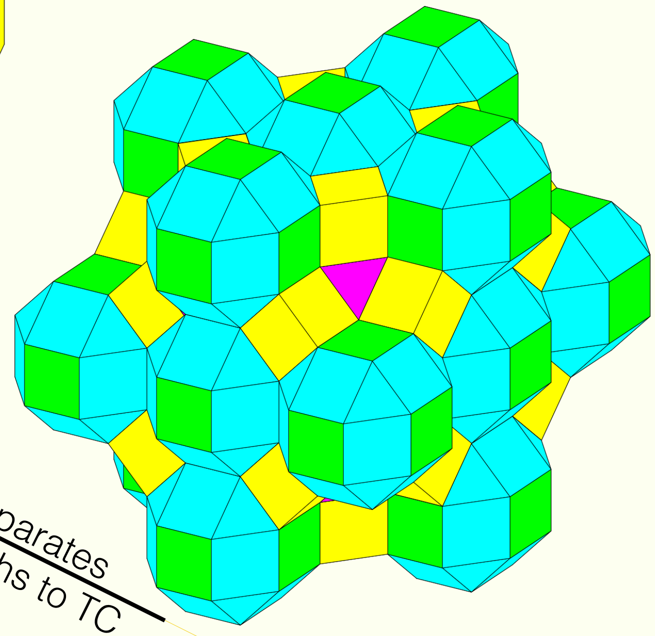
Class III
{2,3,4|2,3,4}
Secondary A

SR: Small Rhombic Cuboctahedron
TC: Truncated Cube
OH: Octahedron
CO: Cuboctahedron
sp: square prism (cube)
op: octagonal prism
ae: axial edge
og: octagon
sq: square
rp: rotated prism (rotated cube)
nv: neutral vertex
rs: rotated square



[SR|TC]
sp|op



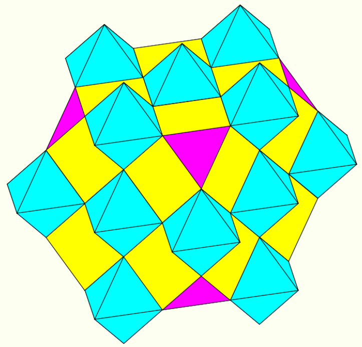


OH morphs to SR
TC separates

SR separates
CO morphs to TC

[OH|TC]
ae|og

[SR|CO]
sq|rp



OH separates
CO morphs to TC

OH morphs to SR
CO separates

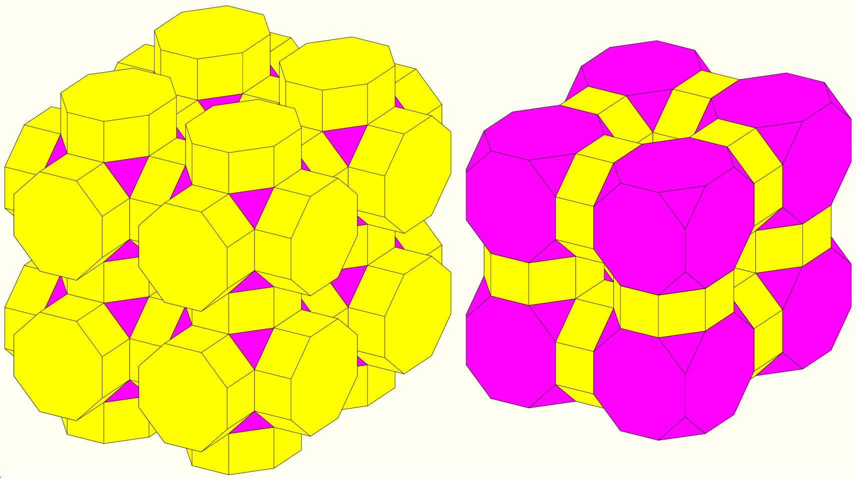
[OH|CO]
nv|rs

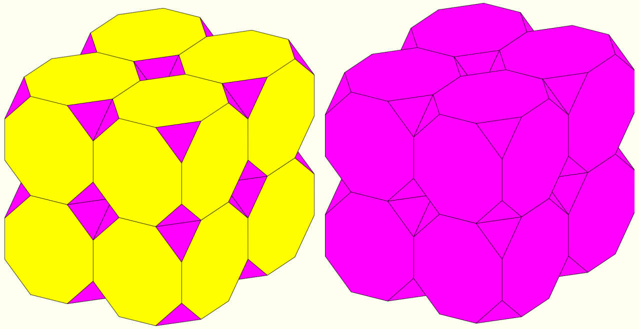
Primary P1
Primary P2
Neutral N1
Neutral N2

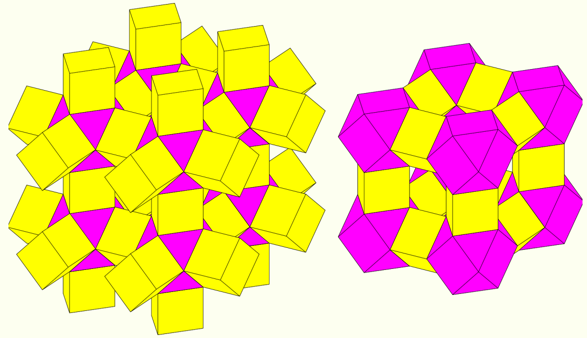
Male + Female

Class III
{2,3,4|2,3,4}
Secondary A

SR: Small Rhombic Cuboctahedron
TC: Truncated Cube
OH: Octahedron
CO: Cuboctahedron
sp: square prism (cube)
op: octagonal prism
ae: axial edge
og: octagon
sq: square
rp: rotated prism (rotated cube)
nv: neutral vertex
rs: rotated square





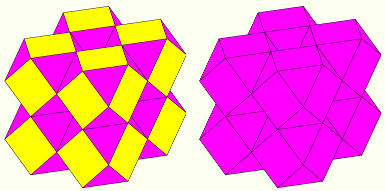


[SR|TC]
sp|op

OH morphs to SR
TC separates

SR separates
CO morphs to TC

[OH|TC]
ae|og



[SR|CO]
sq|rp

OH separates
CO morphs to TC

OH morphs to SR
CO separates

[OH|CO]
nv|rs

Primary P1 | Neutral N1
Primary P2 | Neutral N2

Female only
(extra NEs at left)

Class III
{2,3,4|2,3,4}
Secondary A

SR: Small Rhombic Cuboctahedron
TC: Truncated Cube
OH: Octahedron
CO: Cuboctahedron
sp: square prism (cube)
op: octagonal prism
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og: octagon
sq: square
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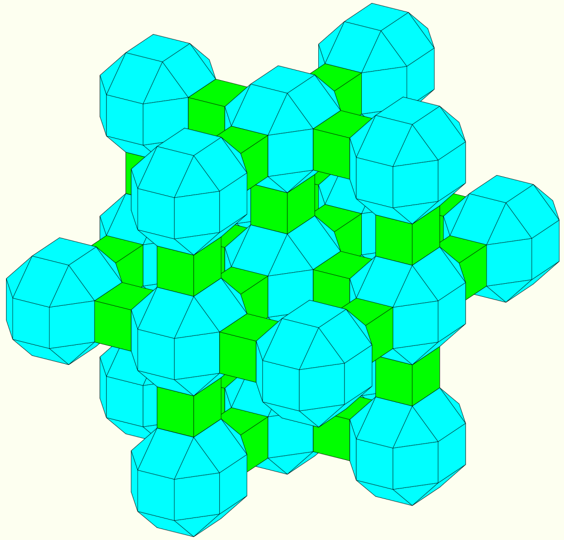
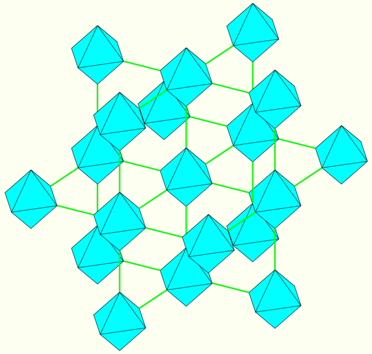
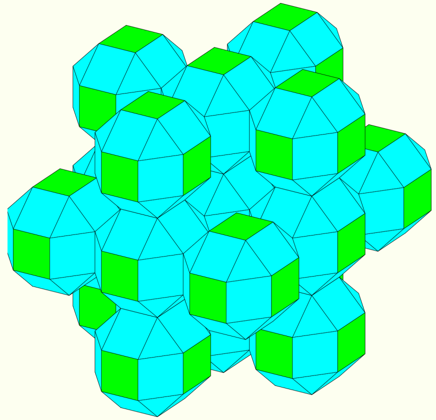




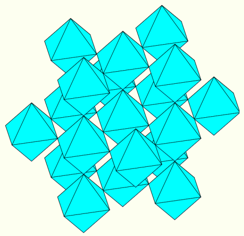


[SR|TC]
sp|op

OH morphs to SR
TC separates

SR separates
CO morphs to TC

[OH|TC]
ae|og



[SR|CO]
sq|rp

OH separates
CO morphs to TC

OH morphs to SR
CO separates

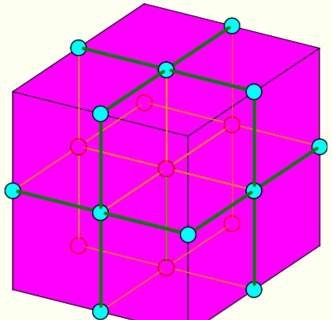
[OH|CO]
nv|rs

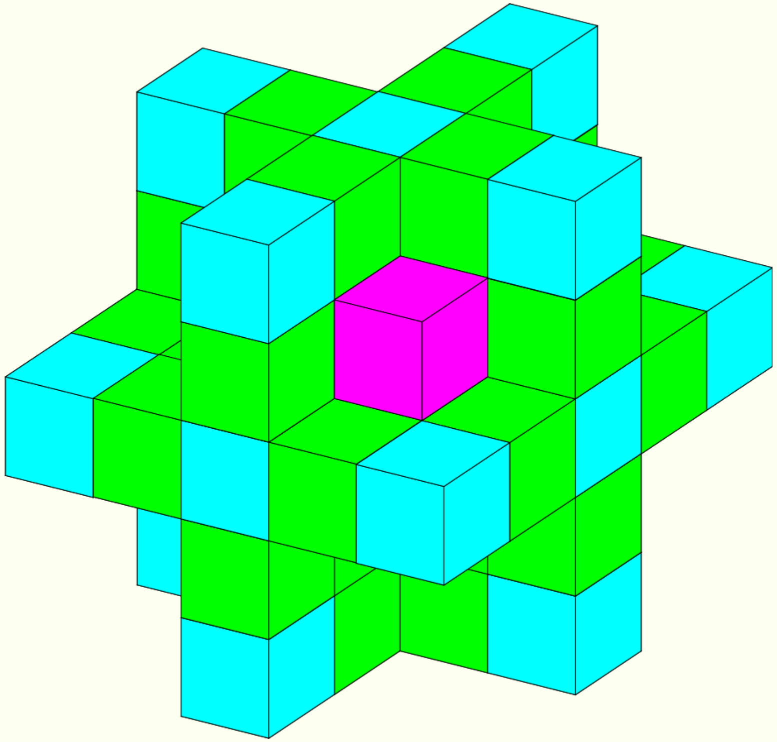
Primary P1
Neutral N1
Primary P2
Neutral N2

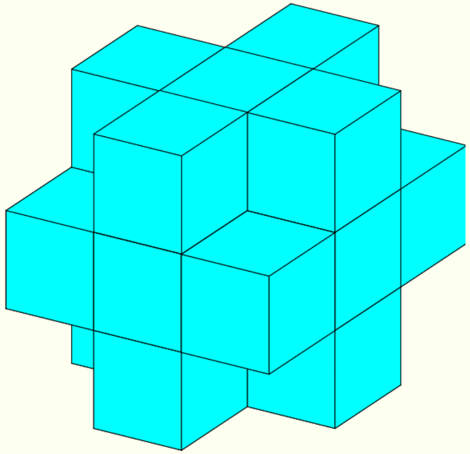
Male only

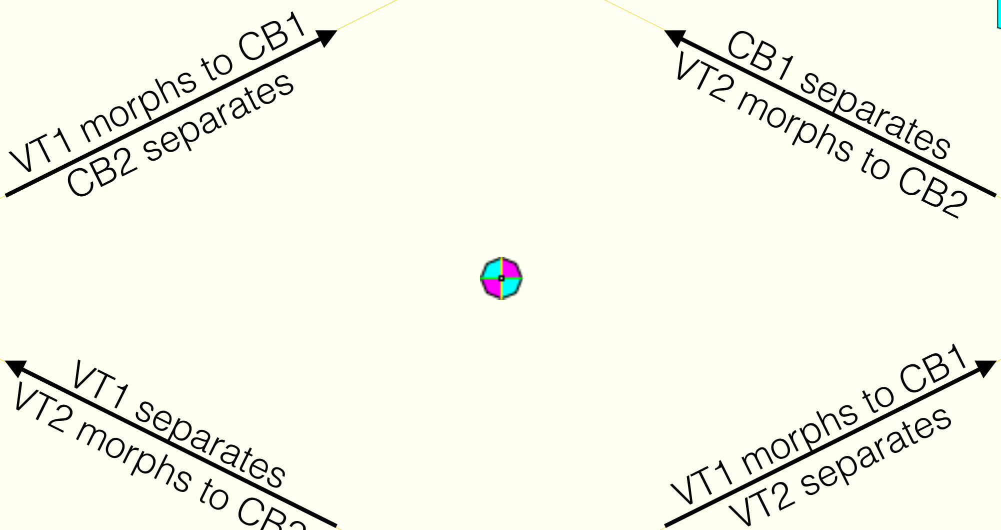
Class III
{2,3,4|2,3,4}
Primary

CB: Cube
VT: Vertex
sp: square prism (cube)
ae: axial edge
nv: neutral vertex







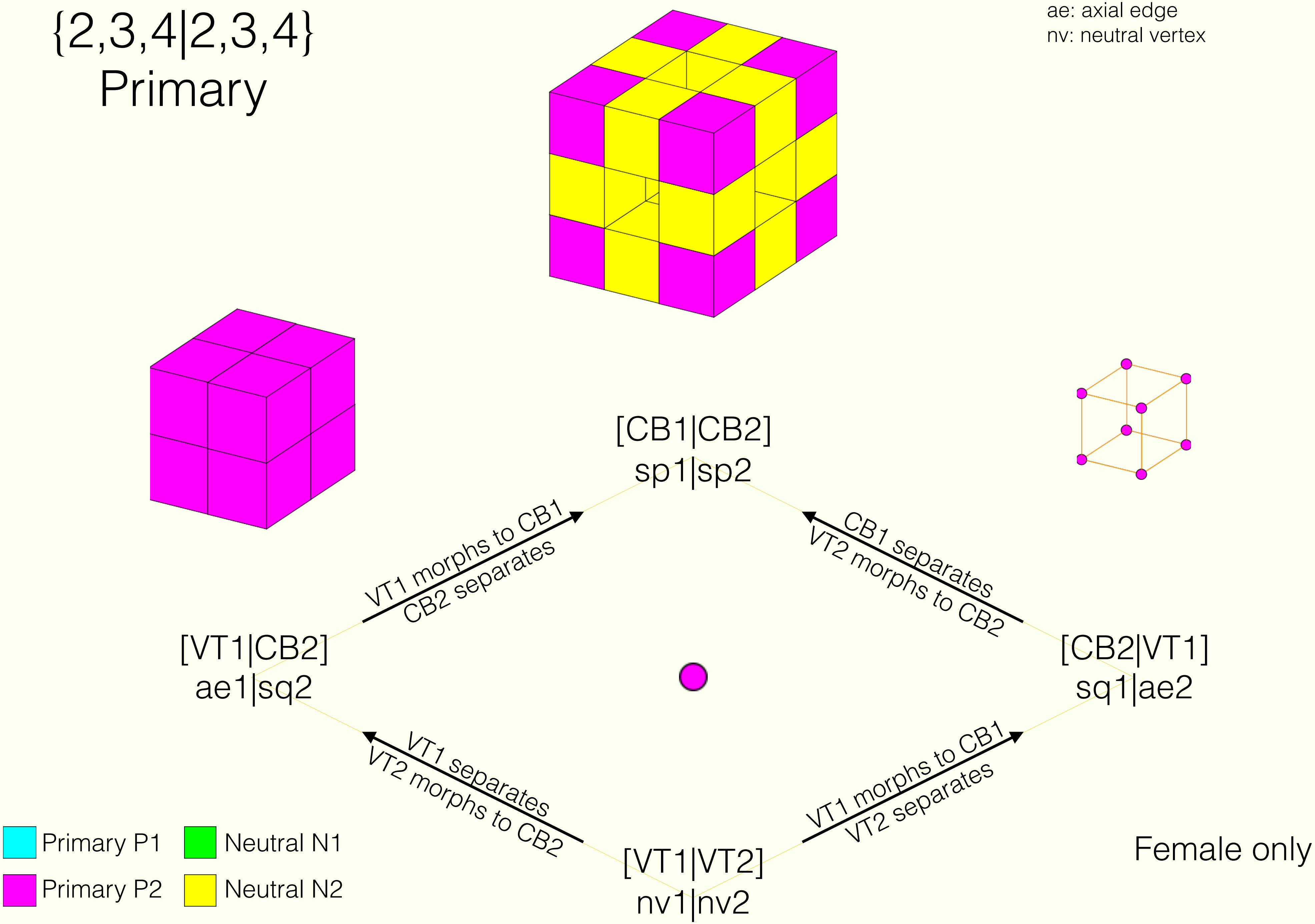


Primary P1 | Neutral N1
Primary P2 | Neutral N2

Male + Female

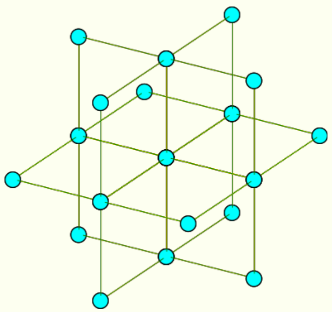
Class III
{2,3,4|2,3,4}
Primary

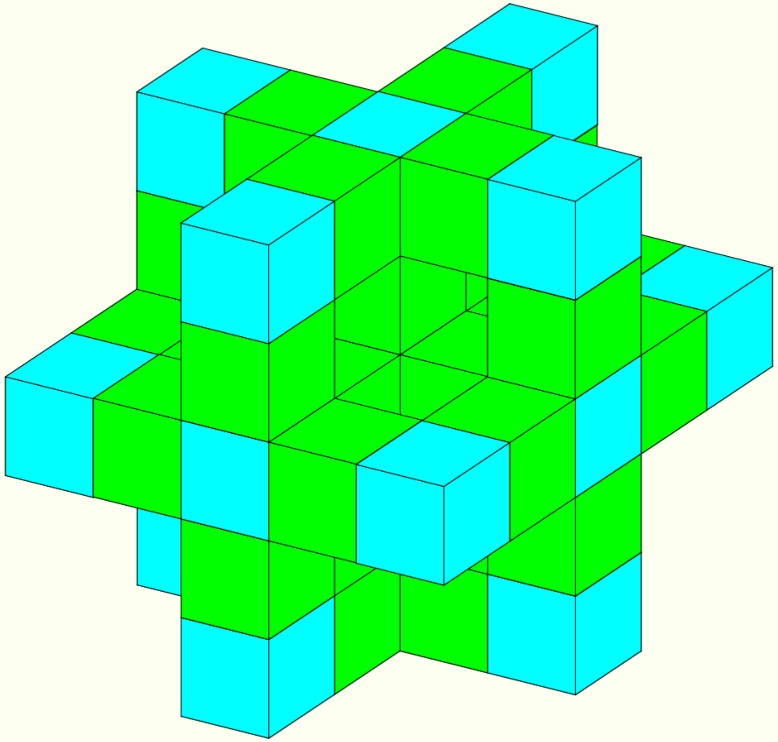
CB: Cube
VT: Vertex
sp: square prism (cube)
ae: axial edge
nv: neutral vertex

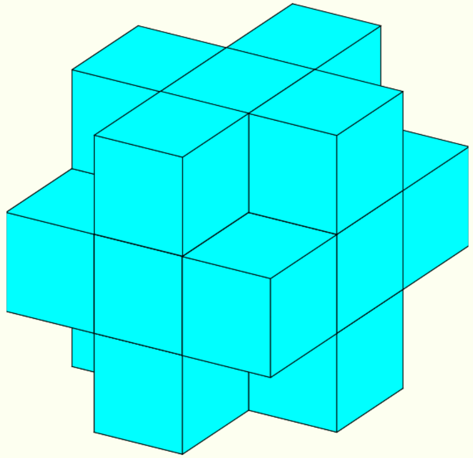


Class III
{2,3,4|2,3,4}
Primary

CB: Cube
VT: Vertex
sp: square prism (cube)
ae: axial edge
nv: neutral vertex







[CB1|CB2]
sp1|sp2

VT1 morphs to CB1
CB2 separates

CB1 separates
VT2 morphs to CB2

[VT1|CB2]
ae1|sq2

[CB2|VT1]
sq1|ae2

VT1 separates
VT2 morphs to CB2

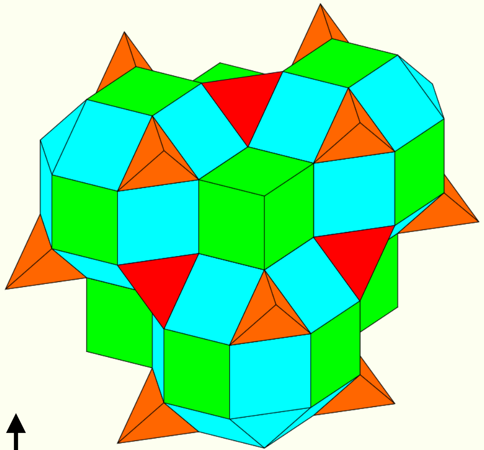
VT1 morphs to CB1
VT2 separates

[VT1|VT2]
nv1|nv2

Primary P1 | Neutral N1
Primary P2 | Neutral N2

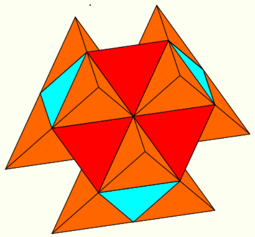
Male only

Class II {2,3,3|2,3,4} Primary & Secondary

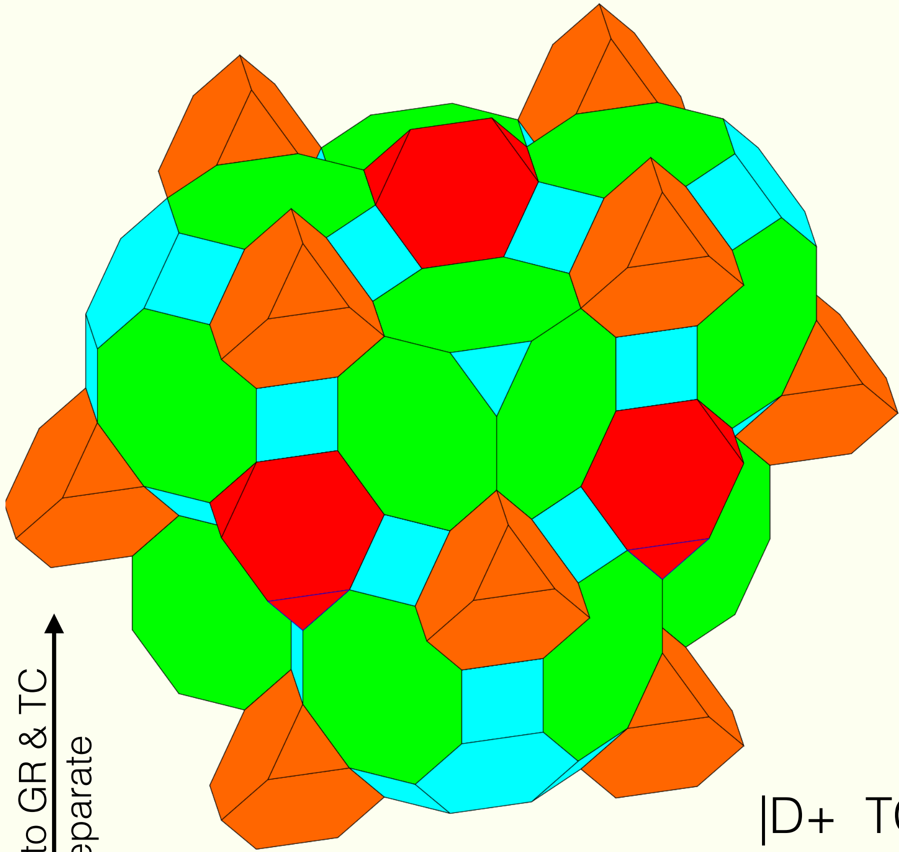


|T+ CB|
|SR T- |

OH & VT morph to SR & CB
T+ and T- separate

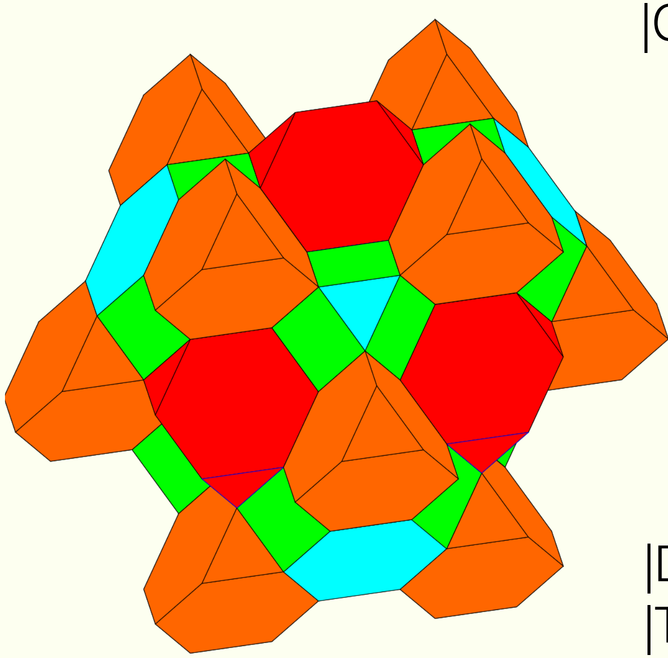


|T+ VT|
|OH T- |

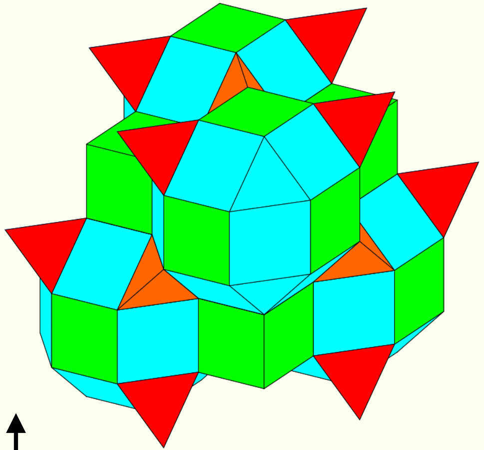


|D+ TC|
|GR D- |

TO & CO morph to GR & TC
D+ and D- separate



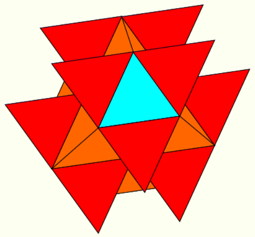
|D+ CO|
|TO D- |



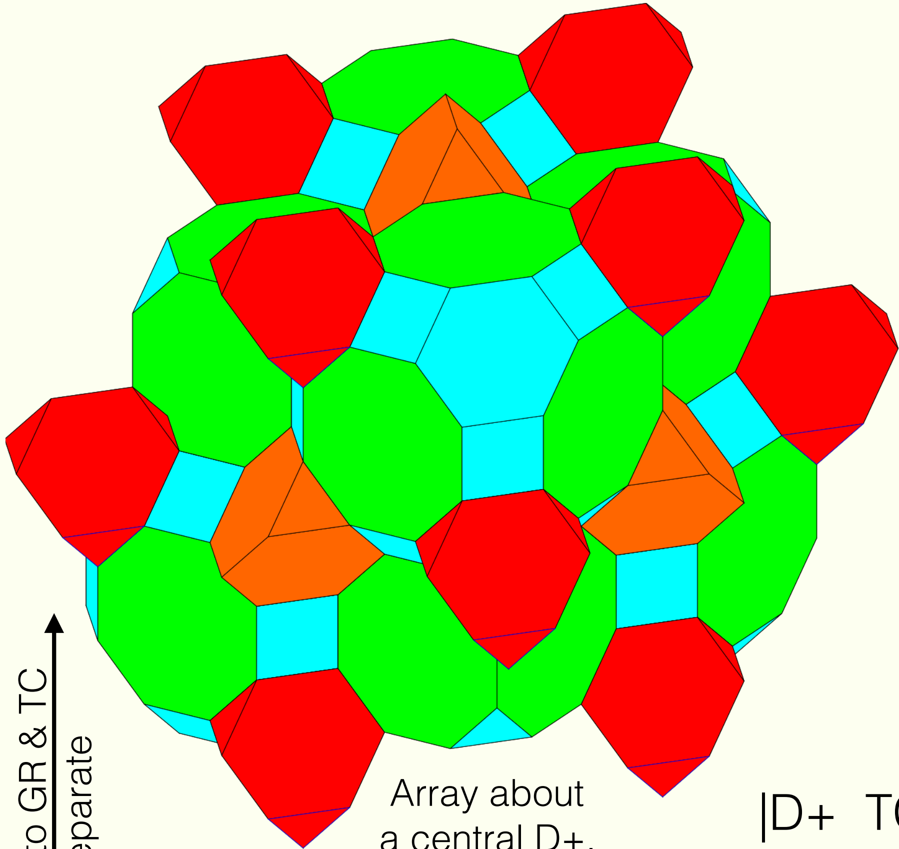
OH & VT morph to SR & CB
T+ and T- separate

Array about
a central T-.

|T+ CB|
|SR T- |



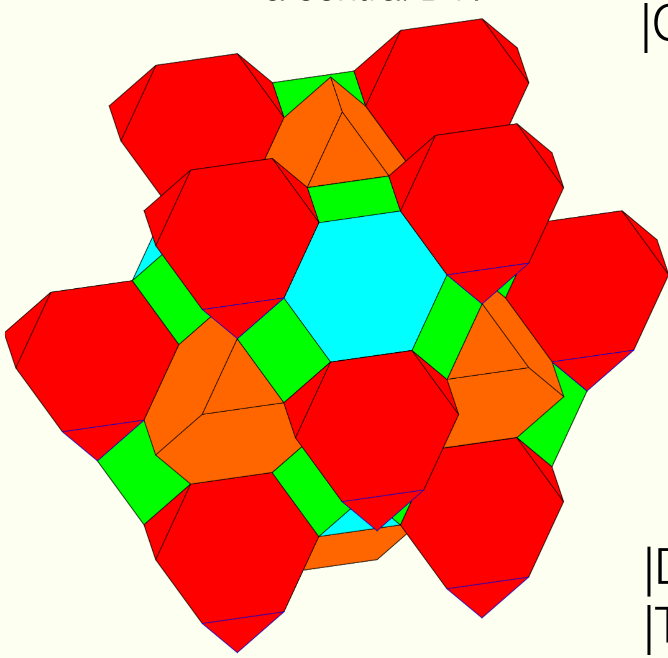
|T+ VT|
|OH T- |



TO & CO morph to GR & TC
D+ and D- separate

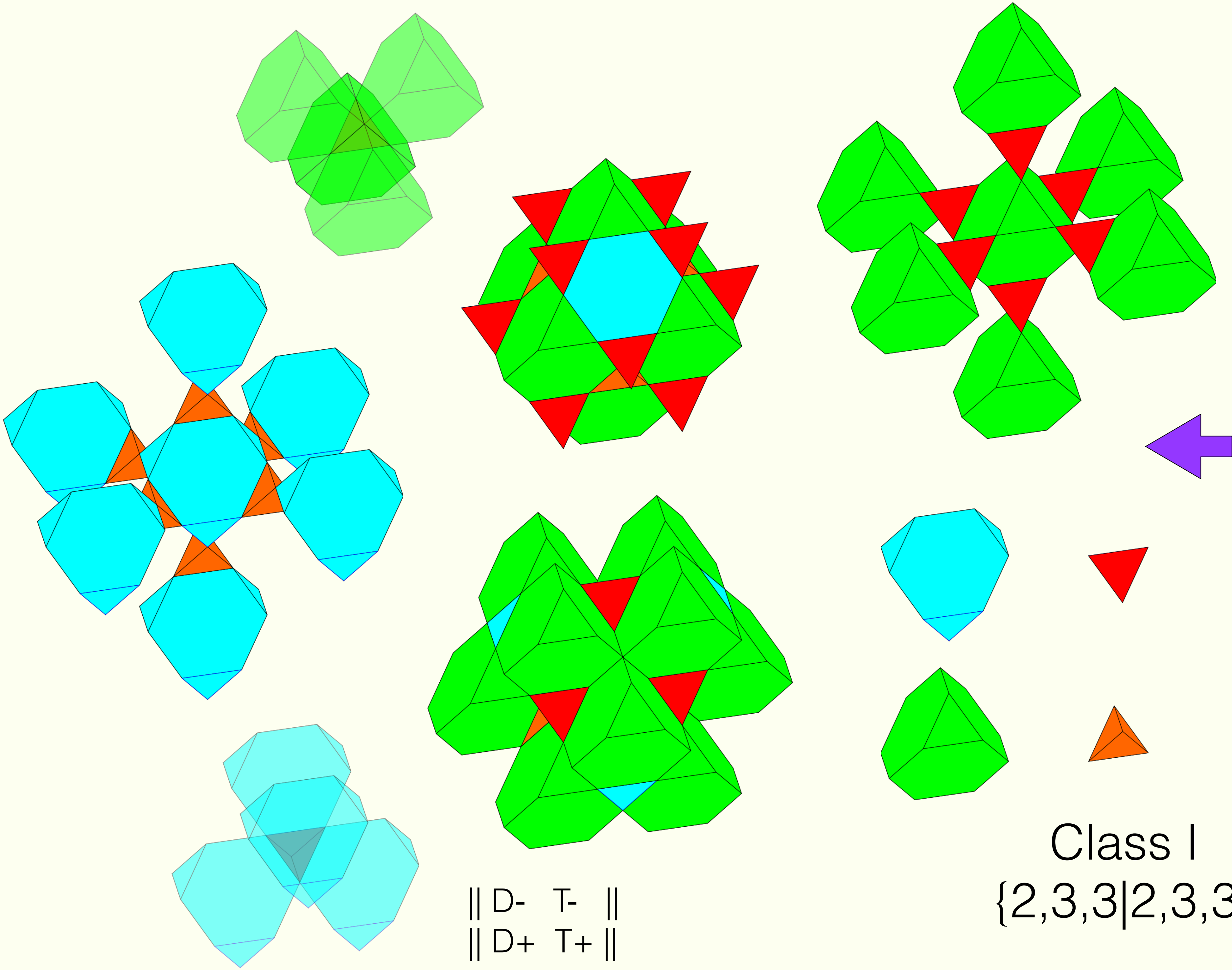
Array about
a central D+.

|D+ TC|
|GR D- |

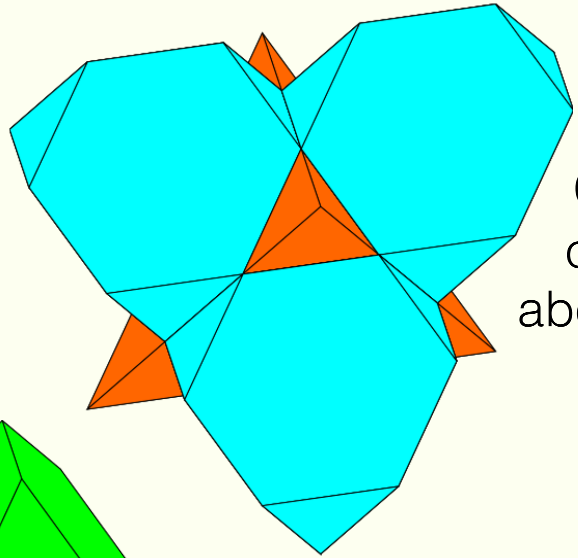


|D+ CO|
|TO D- |

Class II
{2,3,3|2,3,4}
Primary & Secondary

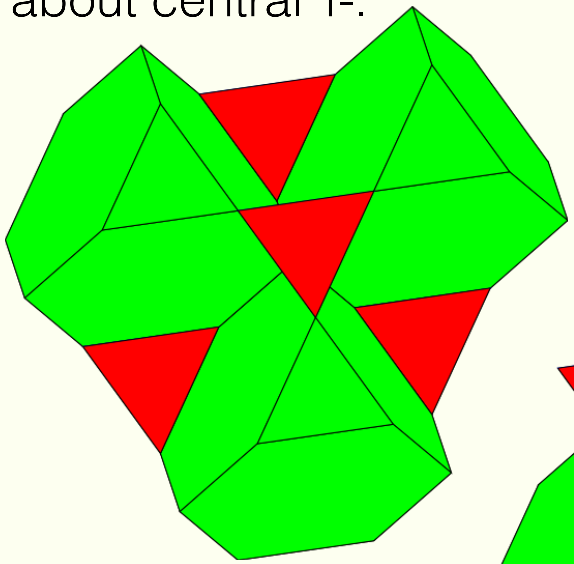


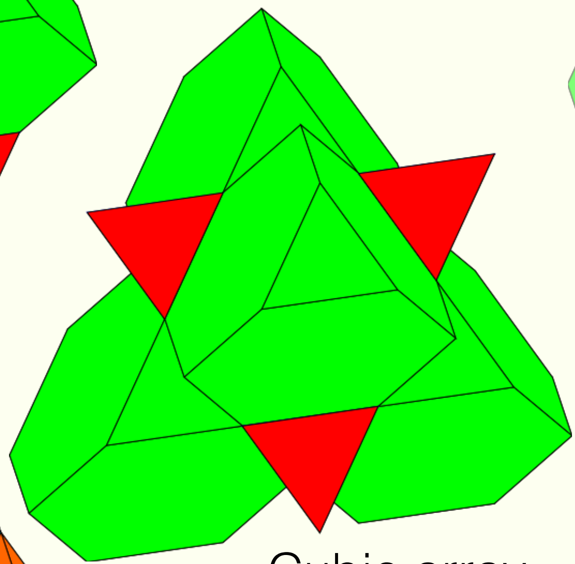
Class I
{2,3,3|2,3,3}



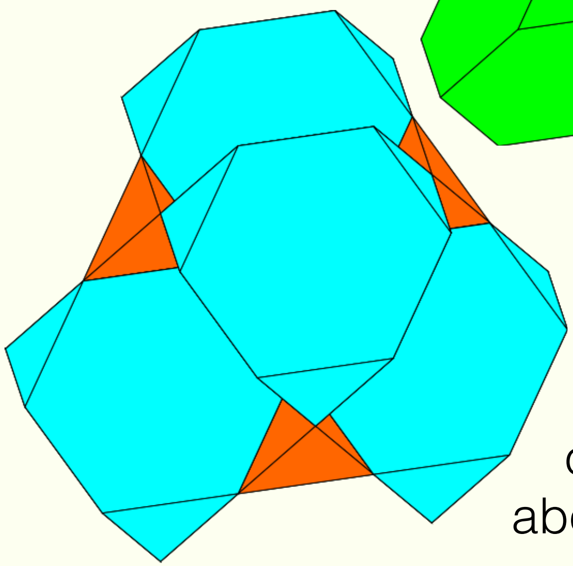
Cubic array
of T- and D+
about central D-.

Cubic array
of T+ and D-
about central T-.

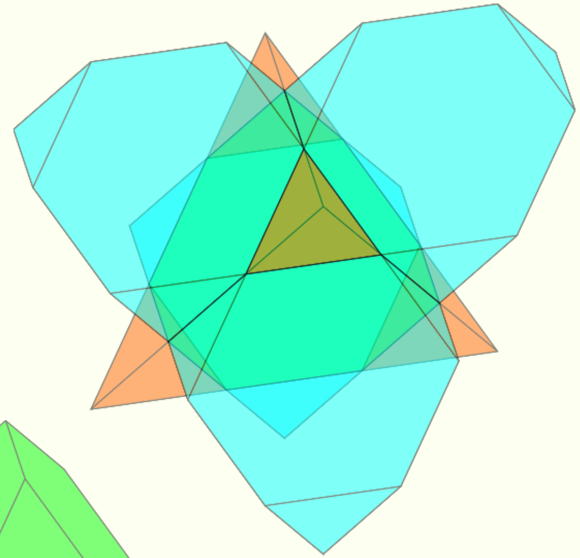


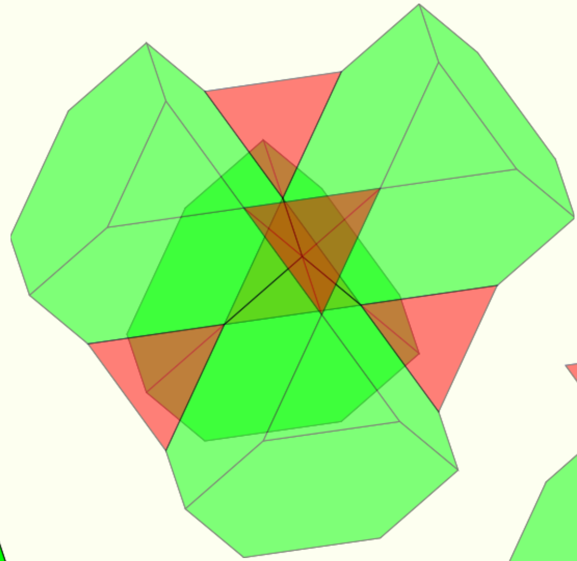


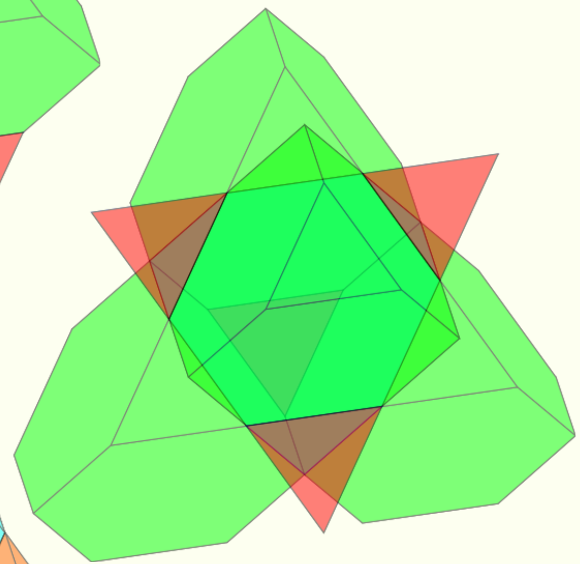
Cubic array
of T+ and D-
about central D+.

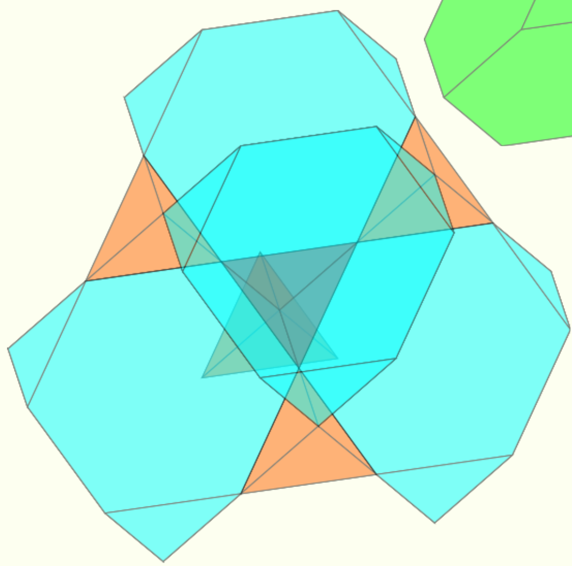


Cubic array
of T- and D+
about central T+.





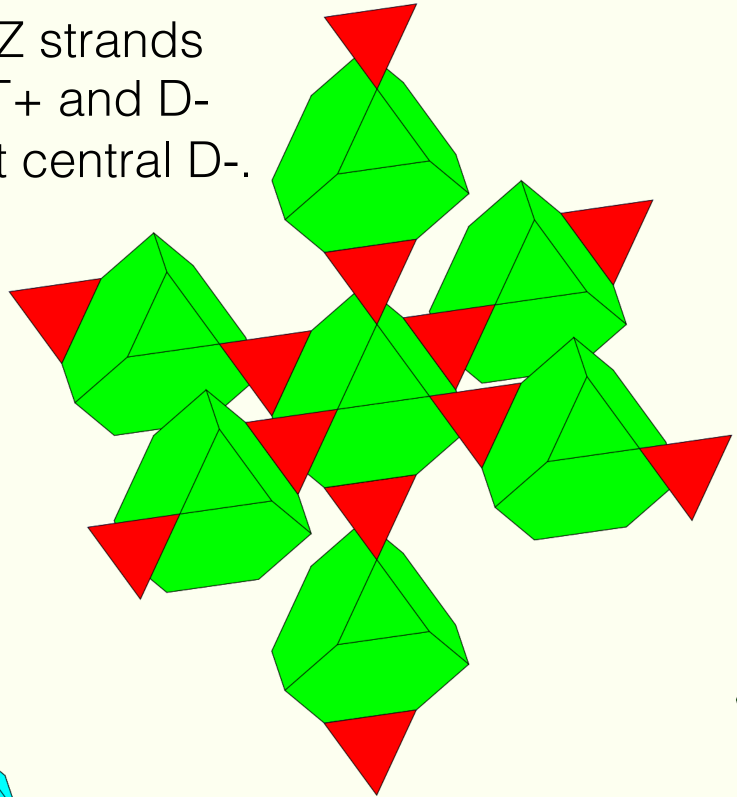




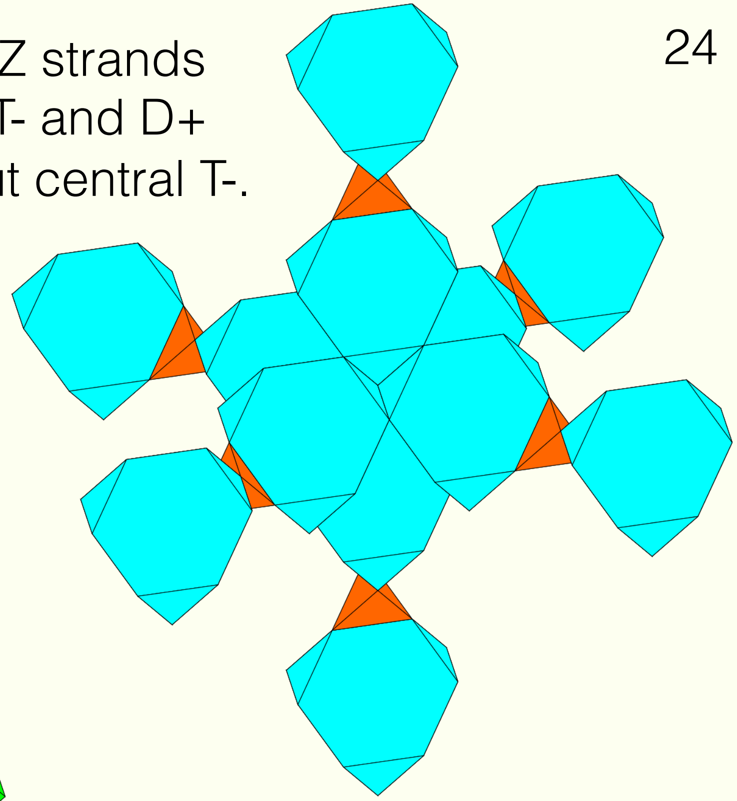
|| D- T- ||
|| D+ T+ ||

Class I
{2,3,3|2,3,3}

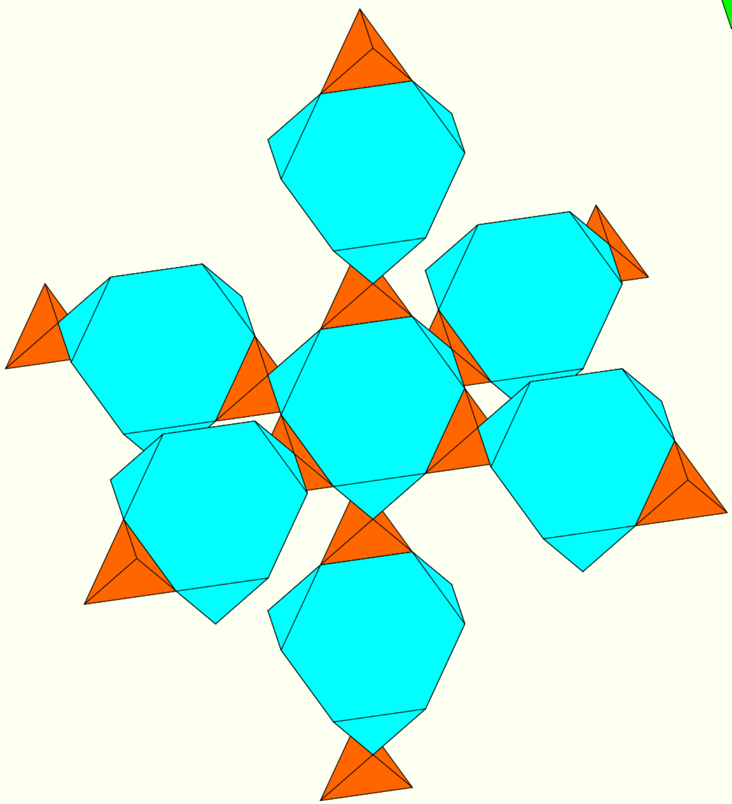
XYZ strands
of T+ and D-
about central D-.

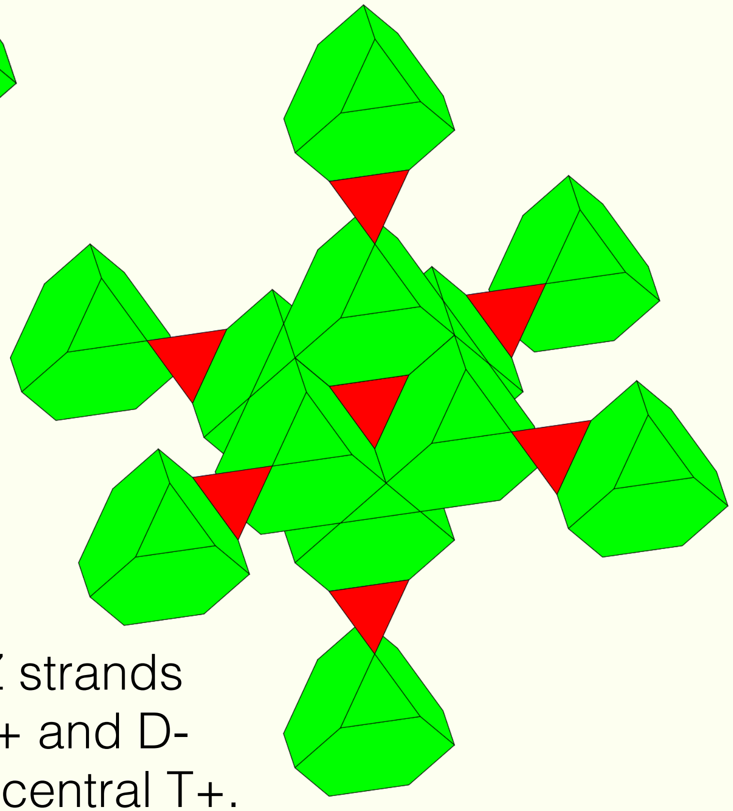


XYZ strands
of T- and D+
about central T-.



XYZ strands
of T- and D+
about central D+.





XYZ strands
of T+ and D-
about central T+.

|| D-   T- ||
|| D+ T+ ||

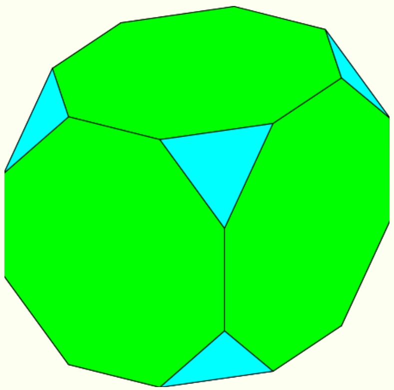
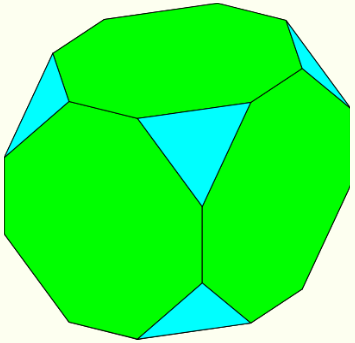
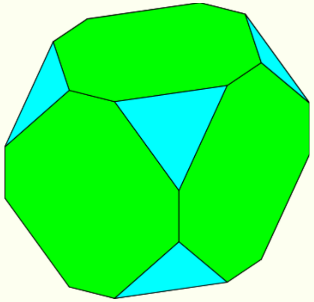
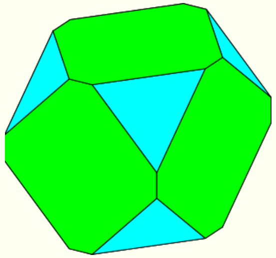
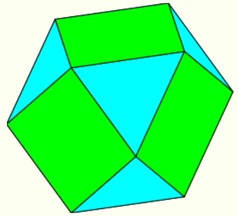
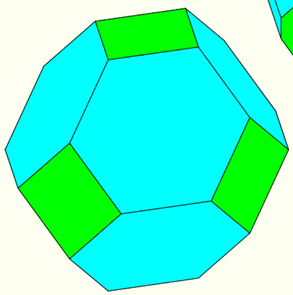
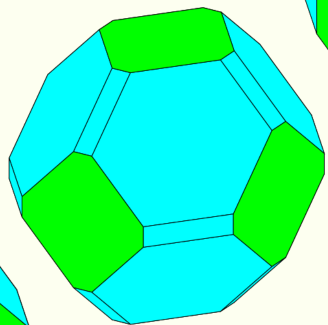
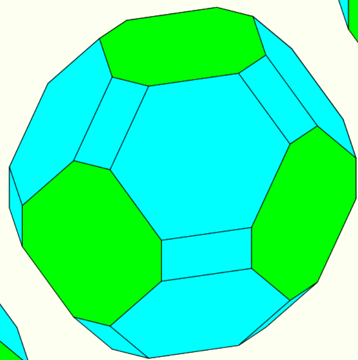
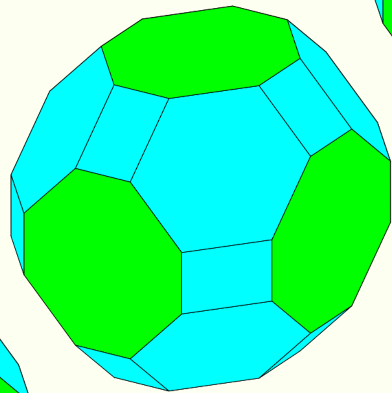
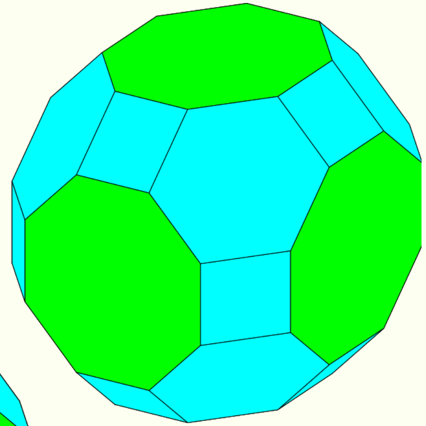
Class I

{2,3,3|2,3,3}

Class II
{2,3,3|2,3,4}

Class III
{2,3,4|2,3,4}

CO morph to TC
rs morph to og

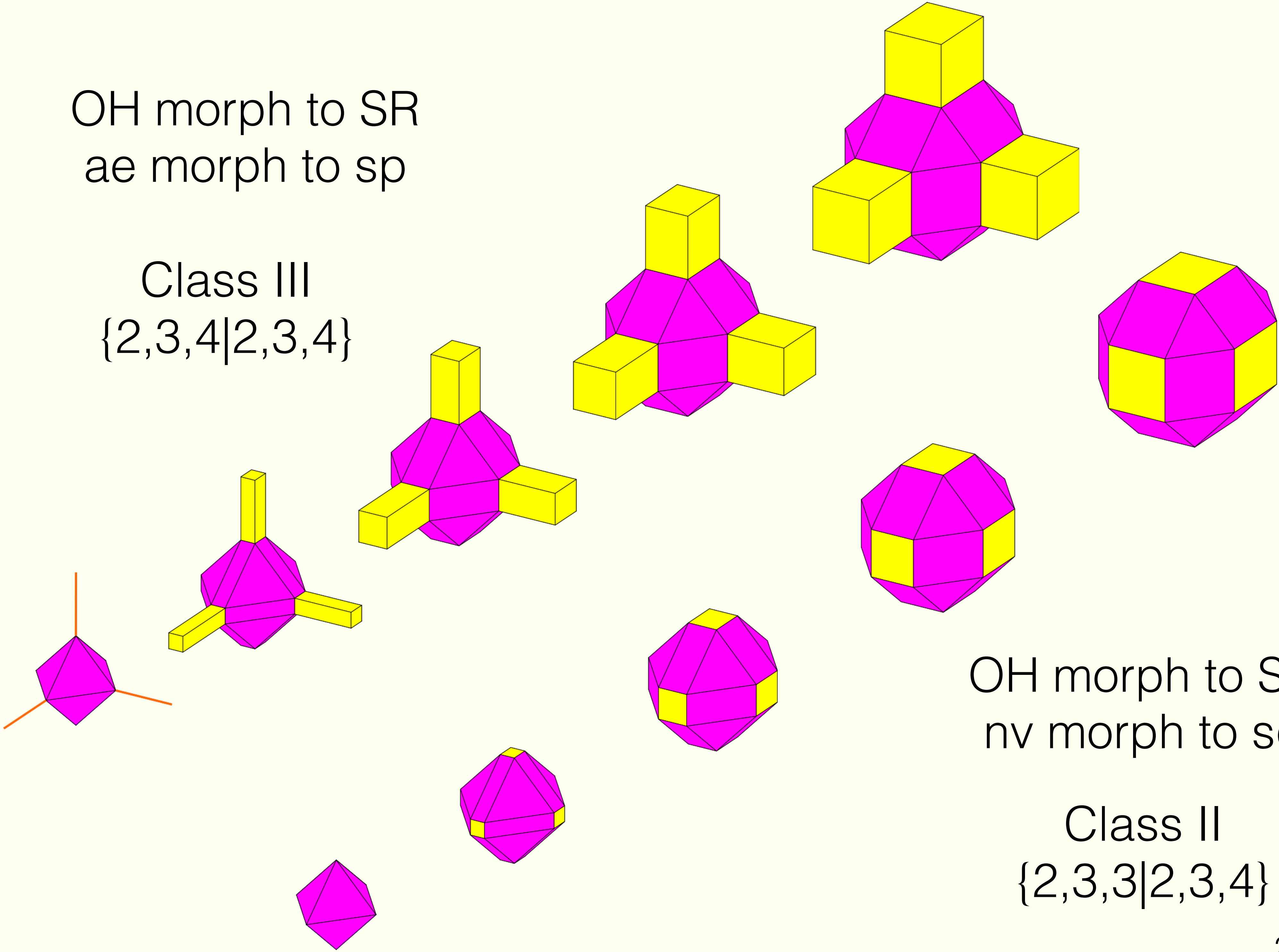
TO morph to GR
rs morph to og

Class II
{2,3,3|2,3,4}

Class III
{2,3,4|2,3,4}

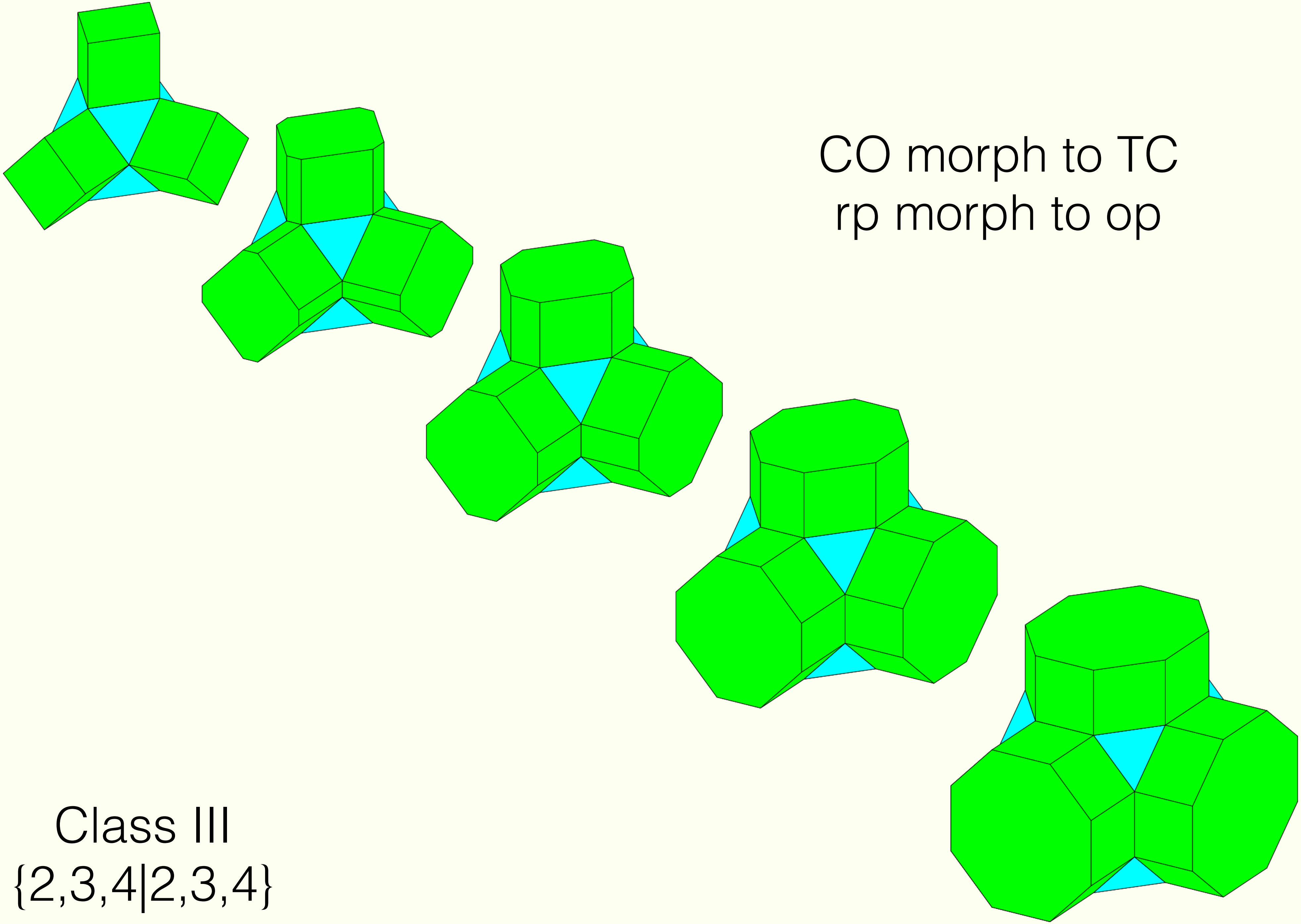
OH morph to SR
ae morph to sp

Class III
{2,3,4|2,3,4}



OH morph to SR
nv morph to sq

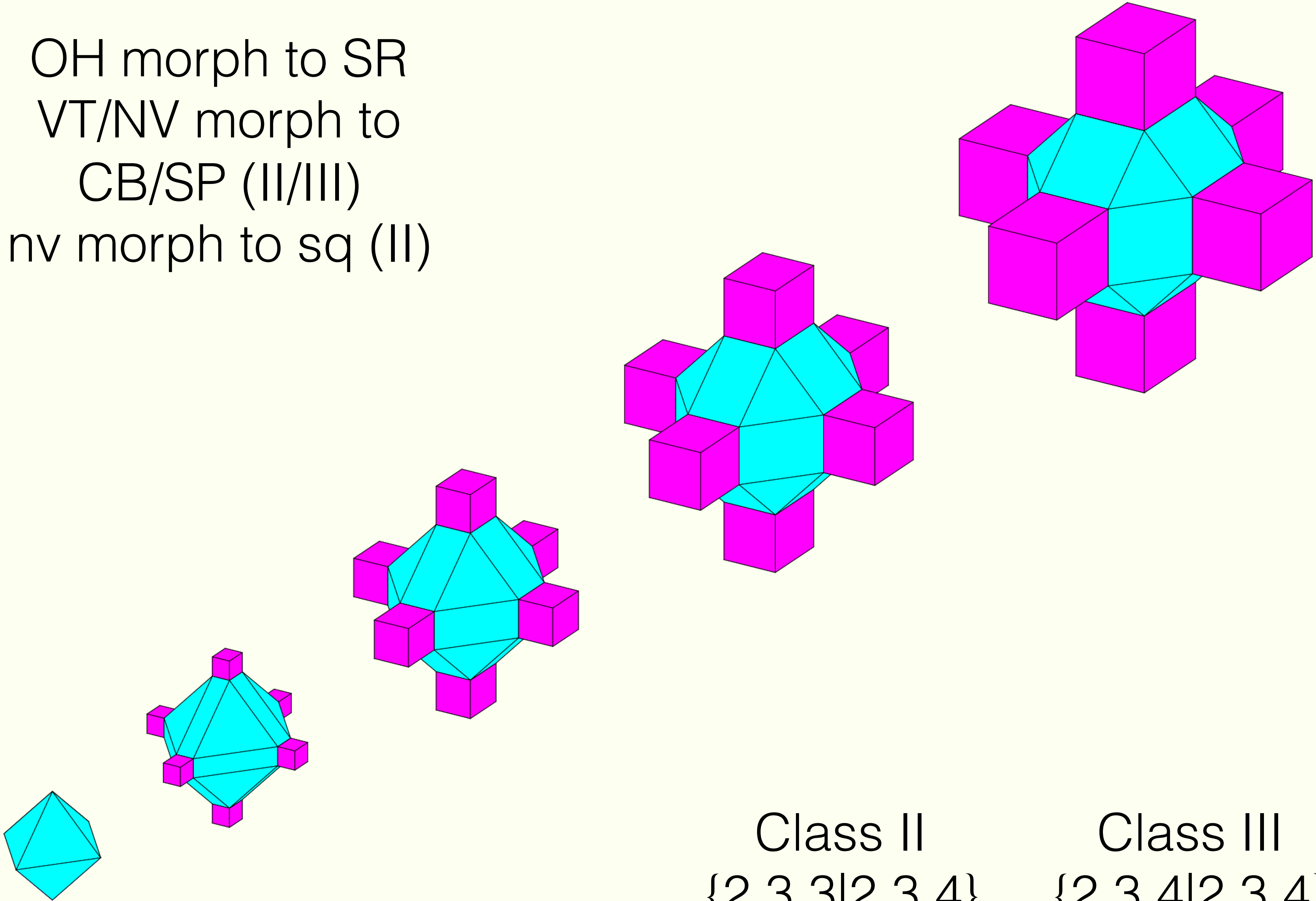
Class II
{2,3,3|2,3,4}



CO morph to TC
rp morph to op

Class III
{2,3,4|2,3,4}

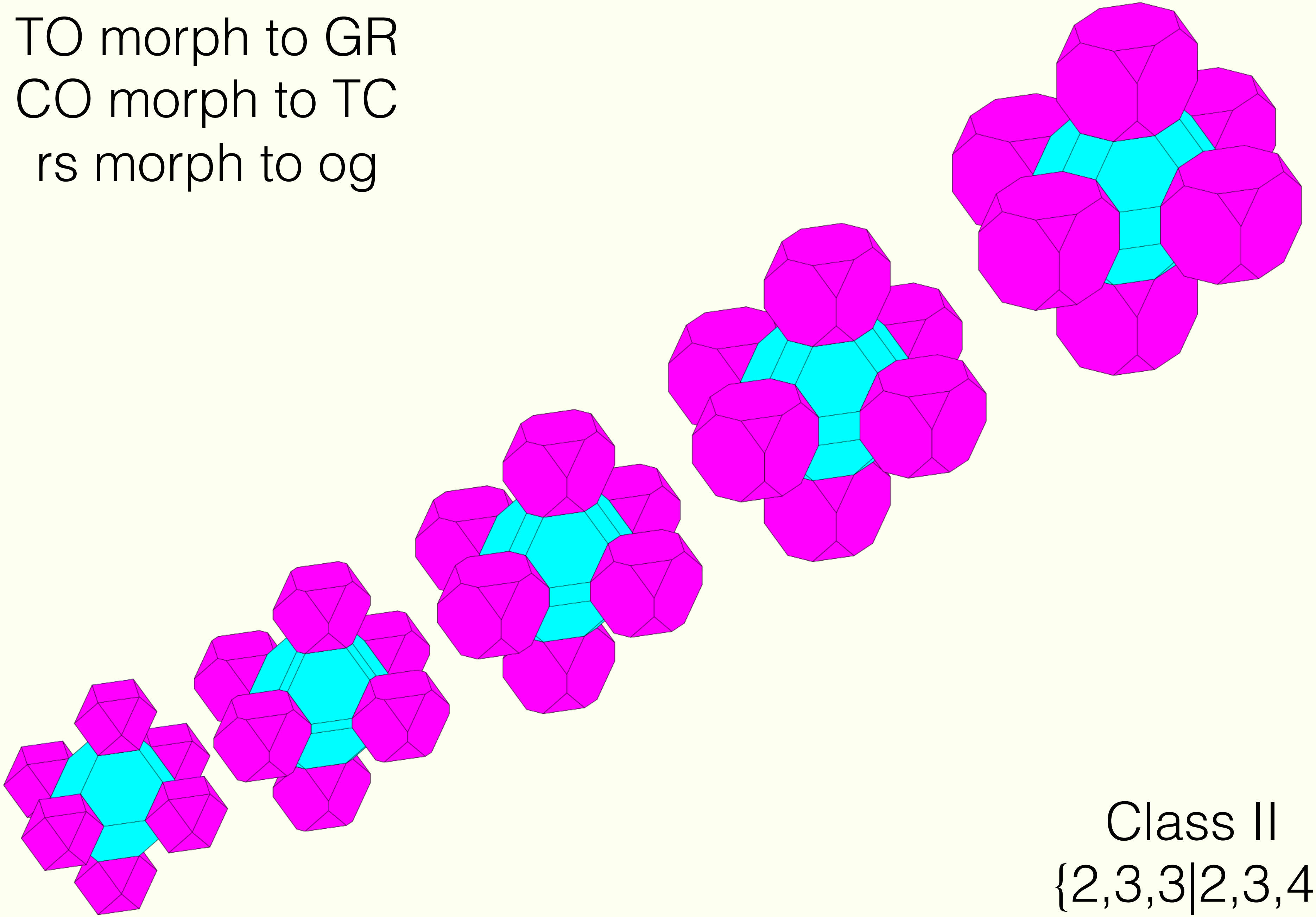
OH morph to SR
VT/NV morph to
CB/SP (II/III)
nv morph to sq (II)



Class II
{2,3,3|2,3,4}

Class III
{2,3,4|2,3,4}

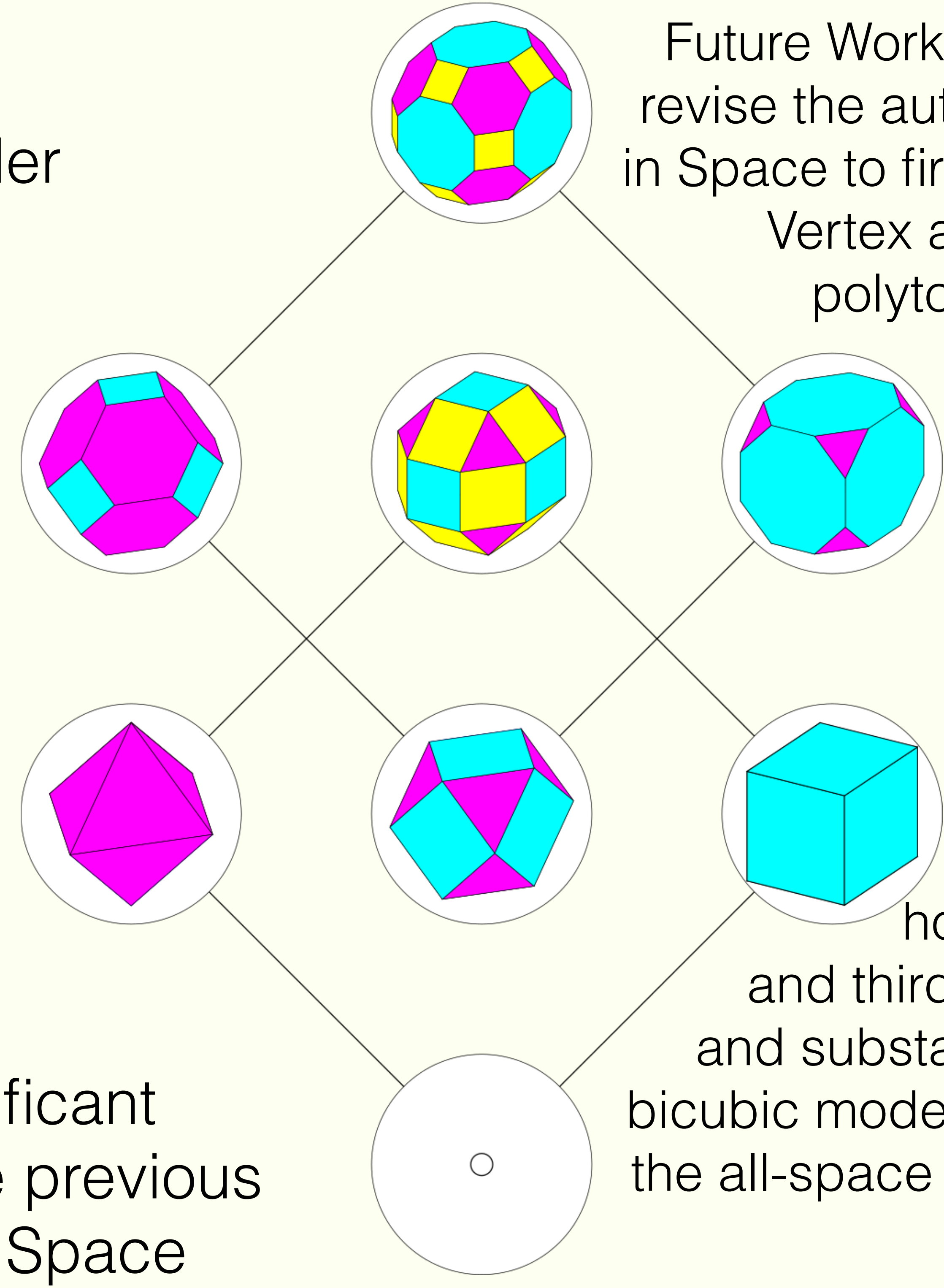
TO morph to GR
CO morph to TC
rs morph to og



Class II
{2,3,3|2,3,4}

A Newer Order in Space

Polyhedral Class II {2,3,4}



Future Work includes significant revision of the previous New Order in Space

Future Work is anticipated to revise the author's New Order in Space to firstly integrate the Vertex as a fundamental polytope in Polyhedral Classes I–V; secondly, clarify the relation between polytopes of one class within that class, and in relation to the polytope honeycomb order; and thirdly, integrate with and substantially revise the bicubic model of metaorder of the all-space filling polyhedral honeycombs.